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Technical Journal of the
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CONTENTS

New laboratories of Creed and Company, Limited, at Croydon, England.

Creed Telegraph Laboratories

THE DEVELOPMENT DIVISION of Creed and Company, Limited, Croydon, England, has recently been transferred to a modern building located about two miles from the company's main offices. The new premises comprise engineering and executives' offices, general laboratory, drawing office, and model shop.

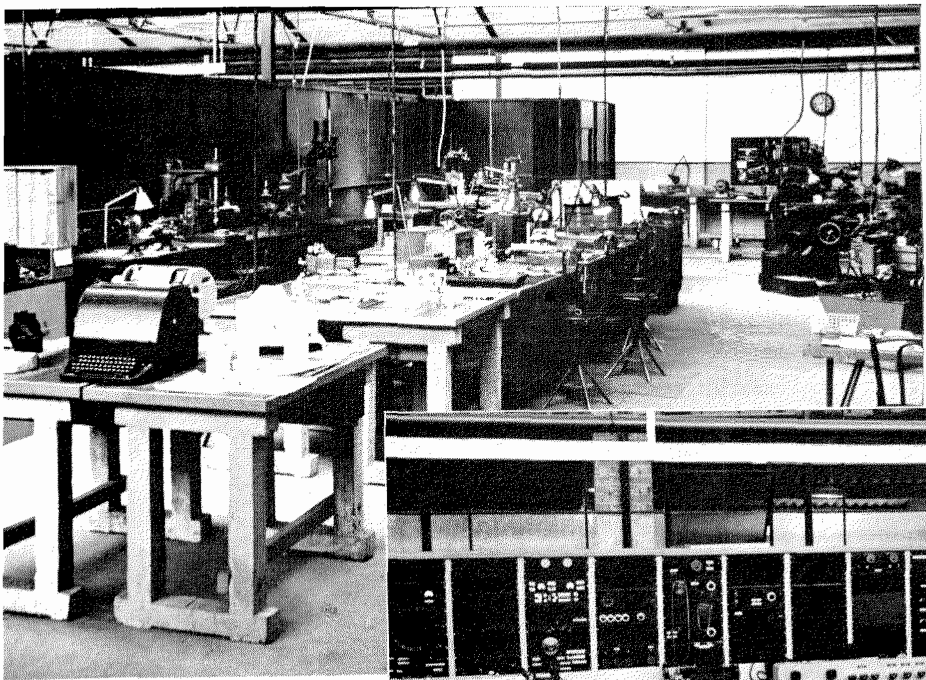
The examination and testing of prototypes of telegraph instruments and mechanisms demand the simulation of much more adverse conditions than are experienced in normal use and require a wide range of facilities, which the new laboratories have been laid out and equipped to provide.

Among the subsidiary laboratories, is a large screened room fully equipped to measure radio interference produced by telegraph apparatus. It is used primarily in the development of suppression devices.

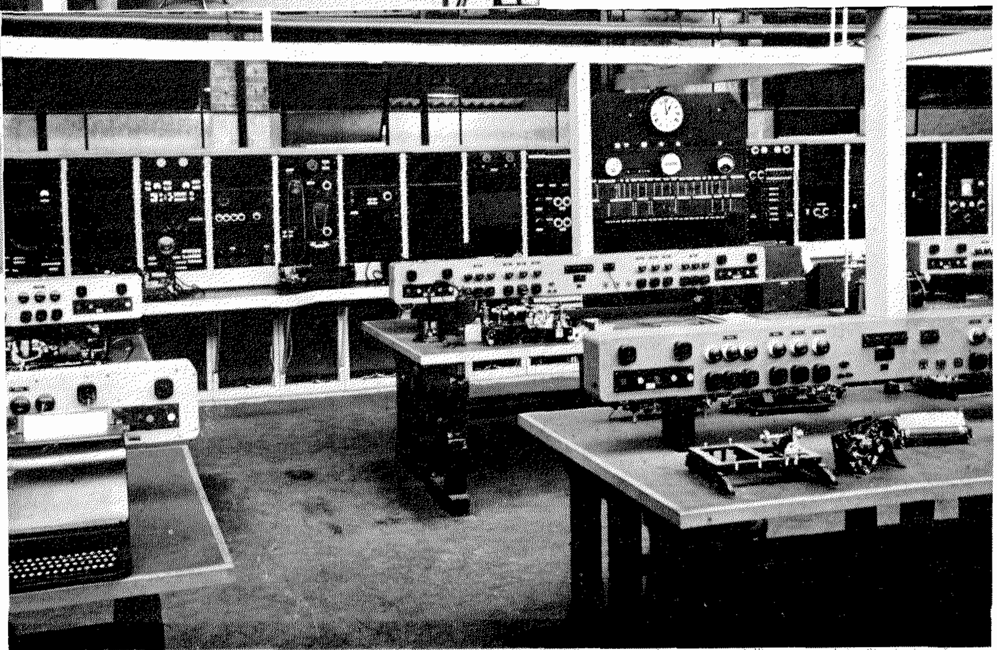
In another room, special provisions have been made for the life testing of apparatus. The equipment incorporates counters that automatically record the actual operating time of the apparatus under test.

Also included, are two rooms with subdued lighting, furnished with specially designed equipment for stroboscopic observation of high-speed mechanisms in actual operation.

The model shop is light and spacious with direct access to the laboratory and the drawing office. It is fully equipped to support the development work in every way.



Model Shop



Main Laboratory

London-Birmingham Television Cable*

By H. STANESBY

British Post Office, London, England

and

W. K. WESTON

Standard Telephones and Cables, Limited, London, England

A CABLE is being laid between London and Birmingham incorporating two 0·975-inch and four 0·375-inch coaxial tubes. The cable is designed to transmit very-high-definition or colour television, 405-line television, and broad-band telephony simultaneously. The large tubes may ultimately transmit frequencies up to 30 megacycles per second or more with repeaters at 3-mile intervals. Details are given of the performance requirements, the cable design, and the results obtained on repeater sections.

• • •

Sound broadcasting depends largely on items that are relayed from one part of the country to another over landlines, and there is little doubt that television broadcasting will evolve in the same way. Moreover, the cost of providing a full and satisfying television programme is so high that as television extends to other parts of the country it will be necessary to transmit major items to as many centres as possible.

All development of the transmission of television signals over long distances stopped in Great Britain between 1939 and 1945 because of the war, and most of the equipment and cables that had previously been used for development work were absorbed in providing additional trunk telephone circuits. Meanwhile, however, ideas were maturing and work was proceeding in the United States, and in 1943, His Majesty's Government appointed a committee under the Chairmanship of Lord Hankey, "To prepare plans for the reinstatement and development of the television service after the war. . ." In its report,[1] published in 1945, this committee recommended that after the war, the 405-line system be reinstated in London and extended first to Birmingham and then to other provincial centres, and that the new stations relay the studio programme from London. It also recommended, among other things, that developments be planned on the assumption that a higher-definition system, perhaps incorporating colour, would for some time be operated side by side with the present system. These recommendations form the framework within which the Post Office has planned the transmission by cable of television signals to Birmingham.†

The link between the existing British Broadcasting Corporation television transmitter and studios at Alexandra Palace, London, and the new transmitter being installed near Sutton Coldfield, Birmingham, will consist of three parts: the main cable between London and Birmingham and a short tail cable at each end. Because it would be uneconomic to provide long-distance wide-band cables exclusively for television, the main cable will form part of the Post Office trunk network and terminate at Museum exchange, London, and Telephone House, Birmingham. Referring to Figure 1, the end connections to the transmitters will be provided by tail cables between Museum exchange and Alexandra Palace and between Telephone House and Sutton Coldfield.

At present all studio items originate at Alexandra Palace; but the British Broadcasting Corporation include in their programmes a considerable number of outside broadcasts, i.e.

* Reprinted from *Post Office Electrical Engineers' Journal*, v. 41, pp. 183–188; January, 1949; and v. 42, pp. 33–38; April, 1949.

[1] Report of the Television Committee 1943; His Majesty's Stationery Office, 1945.

† The Postmaster-General has more recently proposed that the present number of lines, 405, should not be altered for a number of years; and has indicated that the development of a substantially improved system, which might include colour, would take several years, and would be prejudiced if the very slight improvement that would result from increasing the number of lines by 100 or 200 were introduced.

"O.B.s," which are transmitted to British Broadcasting Corporation premises by the Post Office over its television outside-broadcast network or over specially equalised telephone pairs, or by the British Broadcasting Corporation over their outside-broadcast radio link. The British Broadcasting Corporation therefore propose to set up a position at Broadcasting House, London, a few hundred yards from Museum exchange, where they can control the routing of programme material, and a similar switching position is being provided at Broad Street, Birmingham, near Telephone House. The Post Office terminals will therefore be connected to the British Broadcasting Corporation switching positions by short cables which can be regarded as providing extensions of the main and tail cables; in other words, there will be three cable links in tandem between the London and Birmingham transmitters: Alexandra Palace—Museum; Museum—Telephone House; Telephone House—Sutton Coldfield; and for engineering purposes the Post Office will control them from Museum and Telephone House while the British Broadcasting Corporation will have access to them at their near-by switching positions and, of course, at the television transmitters. Museum and Telephone House will also be the terminals of a radio circuit that is being provided as an alternative, 405-line television, link between London and Birmingham;[2] and the tail cables, including the short extensions to Broadcasting House and Broad Street, will be used with the radio link or the main cable as required.

[2] "London-Birmingham Television Radio Relay System," *Post Office Electrical Engineers' Journal*, v. 41, pp. 111–112; July, 1948.

Initially, the cable system will transmit vision signals of 405-line definition only and the video bandwidth will be approximately 3 megacycles per second, nearly 50 per cent greater than is necessary for equal horizontal and vertical definition on the picture,† but in accordance with the 1943 Television Committee's recommendation the whole system is being planned so that it can ultimately be equipped to handle very-high-definition monochrome or colour television at the same time as 405-line signals.

It seems likely that for some time most programme material will pass from Alexandra Palace or the London outside-broadcast cable network to Birmingham; but items will no doubt be transmitted in the other direction to an increasing extent as television becomes more firmly established in other parts of the country; the cables from Telephone House to Museum and Alexandra Palace will therefore provide for two-way transmission.

1. General System Requirements

1.1 Number of Television Tubes

The cable system must ultimately be capable of transmitting 405-line and high-definition

† On the assumption that the image on a television screen of a very narrow nearly horizontal line, has an average width of 1·4 times the line spacing (see H. A. Wheeler, "Fine Structure of Television Images," *Proceedings of the I.R.E.*, v. 26, pp. 540–575; May, 1938).

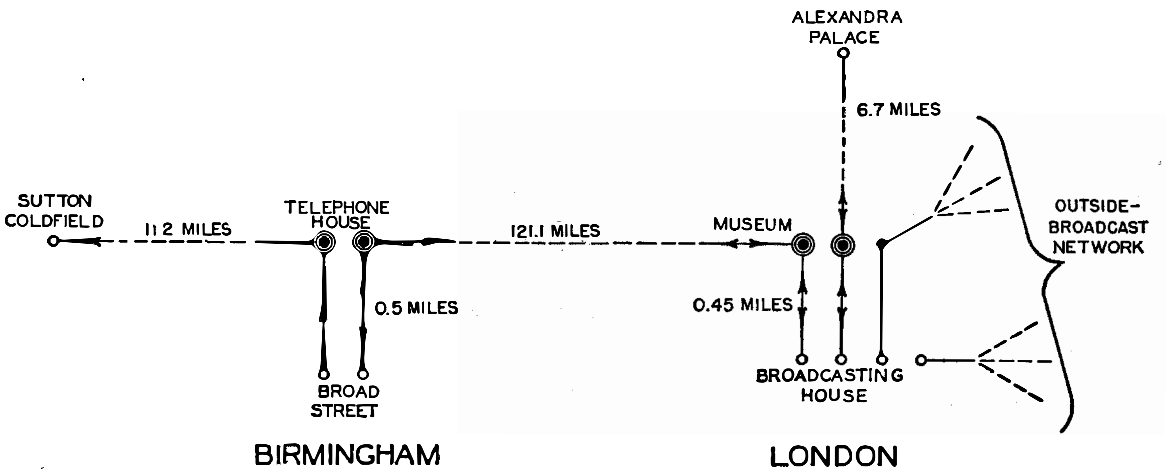


Figure 1—General arrangement of London-Birmingham television cable and tail cables.

signals simultaneously. There is fairly general agreement that a video-frequency band-width of the order of 3 megacycles is adequate for 405-line 50-frame per second television; but for very-high-definition television, or colour television, bandwidths of 10 megacycles or more will be necessary. Up to the present, limitations in valves have made it very difficult to design repeaters for such wide bands, free from harmonic and intermodulation distortion, and sufficiently immune to the effects of supply variations, valve ageing, etc.; and the difficulty would be increased if attempts were made to transmit low- and high-definition signals alongside each other over the same conductors. Separate pairs are, therefore, being provided for the two sets of signals. Coaxial tubes, i.e. coaxial pairs, are used for the same reason that led to their adoption for wide-band carrier telephony: because the frequency at which the loss rises to a given value per unit length is roughly twice as high on a coaxial tube as it is on a balanced pair of the same cross-section, and because the design of the line equipment is much simpler if unbalanced circuits are used, with terminal equipment of a type that makes it unnecessary to transmit very low frequencies along the cable.

Except for the tail cable to the Birmingham transmitter the system will provide for simultaneous transmission in both directions. There are objections to using the same tube for both directions, either (A) simultaneously, by using half the frequency band for each direction, or (B) on a reversible basis by switching line and terminal equipment. Alternative (A) would involve the use of filters to separate the "go" and "return" channels, which would introduce prohibitive delay distortion, and (B) would be complicated, and would lack operating flexibility. Two tubes are therefore being provided for 405-line television and two for high definition.

1.2 Size of Television Tubes

The most economical method of transmitting television signals over coaxial cable is by the asymmetric- or vestigial-sideband system, which needs a bandwidth rather wider than the video-frequency band. For a reasonable compromise between the various factors involved it is perhaps 15 per cent larger. This means that the channels used for transmitting 405-line television will be called upon to handle a band of approximately 3·5 megacycles. This can just be accommodated comfortably on coaxial tubes in which the outer conductor has an inside diameter of 0·375 inch, with repeaters at 6-mile intervals, transmitting frequencies up to 4 megacycles; indeed, the Comité Consultatif International Téléphonique have recently recommended that a European broad-band system be planned on such a basis. The 405-line television tubes will therefore be of this size.

It has not been so easy to determine the best diameter and the other requirements for the high-definition tubes because, of course, the precise form of the signals that will ultimately be transmitted is not yet known. However, there is no doubt that wide bands will have to be handled, and frequencies up to perhaps 30 or even 40 megacycles may be employed, which makes it desirable to use as large a tube as possible to reduce attenuation. Bearing in mind the size limitation imposed by Post Office ducts* and the need to accommodate two high-definition tubes and two 405-line television tubes in the cable, it appeared that the former should have an overall diameter of about 1 inch.

To confirm that such a tube diameter would be satisfactory, certain assumptions have been made about the signals that will be transmitted and the results that could be obtained on very-wide-band repeaters with present-day valves. It has been assumed that:

A. A video-frequency band of 12 megacycles is desirable, which, with a vestigial-sideband system, leads to a cable bandwidth of about 14 megacycles.

B. It is difficult to avoid appreciable second-order modulation in repeaters covering very wide bands, which makes it desirable to avoid bands much exceeding a 2:1 frequency ratio, i.e. say, 12–26 megacycles for a 14-megacycle bandwidth.

C. With present-day technique it would be very inconvenient to employ repeater sections with a loss at 26 megacycles much above 30 decibels; this is because the sending level would be limited by the overload point of the repeaters and the receiving level should not fall to the point at which noise becomes troublesome.

D. The repeater spacing should be equal to, or a sub-multiple of, that used for the 0·375-inch tubes, i.e. $6/N$ miles where N is an integer.

* The internal diameter of Post Office multiple-way ducts used on main trunk routes is $3\frac{1}{4}$ inches, and it is desirable to limit the overall diameter of trunk cables to $2\frac{3}{4}$ inches.

The attenuation A of a coaxial tube of the type used for long-distance transmission is given approximately by:

$$A = 1{\cdot}52/f^{\frac{1}{2}}/D \textit{ decibels per mile}, \qquad (1)$$

where f is the frequency in megacycles and D is the inner diameter of the outer conductor in inches. If D is just less than 1 inch, say, 0·975 inch, which is in fact the value that has been adopted, the loss at 26 megacycles will be 8 decibels per mile, i.e. 48 decibels for the 6-mile repeater sections planned for the 0·375-inch tubes. Requirements C and D, above, therefore indicate that a nominal 3-mile spacing would be needed for the high-definition tubes, corresponding to a loss of approximately 24 decibels at 26 megacycles. As it is desirable to leave margins for (A) some longer sections where there is difficulty in finding repeater station sites, (B) the basic loss of equalisers, 24 decibels is a convenient figure; it is moreover one that is convenient from the point of view of repeater design. The large tubes therefore have an outer conductor with an inner diameter of 0·975 inch, and provision is being made for repeaters at 3-mile intervals.

1.3 Cable Lay–Up

In considering the cable lay-up it became apparent that two more 0·375-inch tubes could be incorporated without affecting the diameter of the complete cable or the large tubes appreciably. As the extra cost is comparatively small they have been included and will be used for a multichannel telephone system. Referring to Figure 2, in which the lay-up is shown, the eight 40-pound quads between the two large tubes will be associated with these tubes for control and supervisory purposes; the two sets of four 40-pound screened pairs will provide ordinary sound broadcast channels as well as those for the television channels; and the other pairs, which are laid up with the 0·375-inch tubes in the same way as they are for standard four-tube coaxial cables, will be used for controlling and supervising the systems operating over the 0·375-inch tubes.

The tail cable between Birmingham and the Sutton Coldfield transmitter needs only provide for one-way transmission of television and therefore contains only two coaxial tubes. To make a practicable lay-up, both are 0·975-inch tubes. The cable also contains eight 40-pound screened pairs, and a number of unscreened 40-pound quads for miscellaneous purposes including control and supervision.

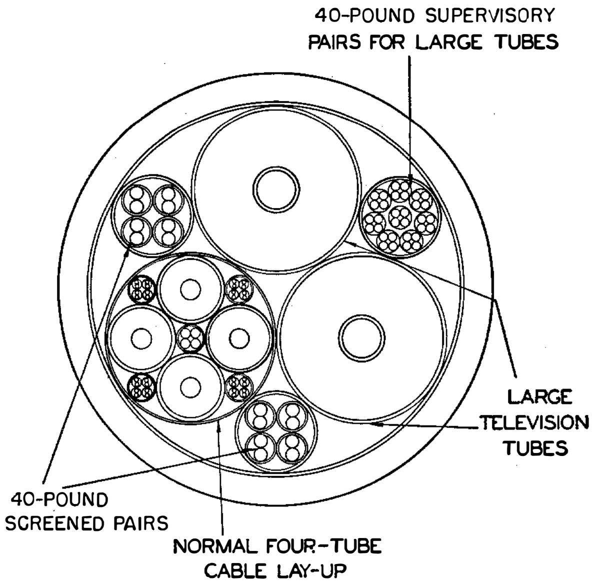


Figure 2—Lay-up of television cable.

2. Electrical Requirements

In considering the other requirements that affect the design and manufacture of the cable, attention will be confined almost entirely to the 0·975-inch coaxial tubes because the smaller tubes and the balanced pairs conform substantially with established practice.

2.1 Characteristic Impedance

For coaxial tubes in which both conductors are of the same material and D the inside diameter of the outer conductor is fixed, the attenuation is a minimum when d the outside diameter of the inner conductor is $D/3{\cdot}59$. When the permittivity of the space between the conductors is unity, i.e. the dielectric is wholly gaseous, this yields a characteristic impedance of 77 ohms. For a permittivity of 1·2, which is on the high side for tubes of the type used in trunk cables, the impedance would be 70 ohms, and to raise it to 75 ohms d would have to be reduced to $D/3{\cdot}94$, which would increase the attenuation by less than 0·5 per cent. Such a minute increase is trivial compared with the advantage of maintaining a standard impedance value. The Post Office has always adhered to 75 ohms as the nominal characteristic impedance for coaxial tubes and has retained that value for the present cable. The specification calls for the characteristic impedance of the large tubes averaged over a small interval centred on 20 megacycles to be between 74 and 76 ohms.

It is inevitable that the characteristic impedance of a coaxial tube or any other form of transmission line will vary slightly from point to point. These variations can occur within drum lengths, from drum length to drum length, or may be introduced by the joints themselves; but unless they are very gradual and only become appreciable over very long lengths of cable they give rise to significant internal reflections. Reflections also arise at the ends of repeater sections unless the terminations match the characteristic impedance accurately. If reflections occur at two or more points in a repeater section, part of the signal energy reflected backwards from a point nearer the receiving end will be reflected forward again at a point nearer the sending end and give rise to an echo that arrives after the main signal. In television this may result in a picture in which the main outlines are followed by faint and distorted ghosts. So many factors are involved in considering these echoes and their effects on a television picture that it must suffice here to indicate the way in which the impedance irregularities are assessed and the limits that have been adopted.

Reflections at the ends of repeater sections, although very difficult to eliminate, are determined by the repeater equipment, but irregularities within the cable must as far as possible be avoided during the manufacture, laying, and jointing of the cable. As viewed from the end of a section, with the other end terminated in its characteristic impedance, each irregularity gives rise to a roughly sinusoidal variation of the input impedance with frequency. The amplitude of this variation is proportional to the irregularity modified by the attenuation of the intervening length of cable, and the net effect of a considerable number of irregularities randomly distributed is to cause the input impedance to vary more or less at random about the mean characteristic impedance. Although perhaps not the best from a theoretical point of view, this variation of input impedance is a convenient index of the variation of characteristic impedance along the cable, and the criterion adopted is that for the large tubes the sum of the six largest variations of the resistive component of the input impedance between 3 and 26 megacycles at each end of a 3-mile repeater length shall not exceed 26·5 ohms.* It has been shown from an analysis of the results on a number of lengths of cable that this limit corresponds to a root-mean-square variation of impedance about the mean value of one per cent.

2.2 Attenuation

The attenuation in nepers A_n per unit length of any transmission line in which the conductor resistance R and dielectric conductance G per unit length are small compared with ωL the series reactance and ωC the shunt susceptance is given by:

$$A_n \doteq \frac{R}{2}\left(\frac{C}{L}\right)^{\frac{1}{2}} + \frac{G}{2}\left(\frac{L}{C}\right)^{\frac{1}{2}}. \qquad (2)$$

At high frequencies, where skin effect is fully developed, R is closely proportional to $f^{\frac{1}{2}}$, and G for any good-quality non-dispersive insulating material is proportional to f; and it can be shown that small variations in the distribution of the inductance and capacitance have negligible effect on the attenuation provided the values averaged over the whole of the length in question are constant. Equation (2) can therefore be rewritten as:

$$A = K_1 f^{\frac{1}{2}} + K_2 f \textit{ decibels per mile}, \qquad (3)$$

where K_1 and K_2 are constants determined by the resistance and conductance loss components

* In determining this quantity the highest and lowest turning points in the impedance characteristic are combined in pairs such that the six largest differences are obtained, using each turning point once only. This index is used rather than the root-mean-square deviation because it is more easily determined.

respectively. To ensure that neither component contributes unduly to the loss and that the loss-versus-frequency characteristic approximates reasonably closely to a given law, on the large tubes, between 1 and 40 megacycles, the attenuation per mile corresponding to 15 degrees centigrade, measured on complete 3-mile repeater sections, is required to be within ±4 per cent of the values given by the following expression

$$A = 1{\cdot}56\, f^{\frac{1}{2}} + 0{\cdot}01\, f \textit{ decibels per mile.} \qquad (4)$$

In general the measurements are made at temperatures differing from 15 degrees centigrade, which can be determined from direct-current resistance measurements, and they are corrected to 15 degrees centigrade, using temperature coefficients of loss determined on drum lengths before laying. To ensure that the cable will not be subject to secular change, the attenuation after one year is required to remain within 1 per cent of the values determined during acceptance tests, due allowance being made for differences in temperature.

2.3 Crosstalk

The crosstalk attenuation between coaxial tubes may be expected to increase rapidly with frequency if, as is usual, there is complete longitudinal continuity of the outer conductors. Thus, although the tubes of the present cable, particularly the 0·975-inch tubes, may ultimately be used over very wide frequency ranges, crosstalk at frequencies above about 1000 kilocycles is unlikely to cause any difficulty and need not be considered here.

It is not possible at this stage to decide exactly how all the 0·375-inch and 0·975-inch tubes will finally be used below 1000 kilocycles; it is nevertheless important that crosstalk shall not be a limiting factor. It is therefore assumed that broad-band telephony, on frequencies of 60 kilocycles and above, will be transmitted over some, at least, of the 0·375-inch tubes, with a repeater spacing of 6 miles, and may even be transmitted over the 0·975-inch tubes with a 12-mile repeater spacing. It is also assumed that 405-line television might be transmitted over either type of tube, with the above repeater spacings, using an asymmetric-sideband system involving frequencies from 300 kilocycles upwards, with the carrier frequency not lower than 500 kilocycles.

Where television and telephone channels pass over tubes in the same cable, crosstalk from the former to the latter is likely to be more serious than in the reverse direction, because of the high energy concentrations at, and relatively near, the television carrier. It is therefore sufficient to consider the requirements in terms of the signal-to-crosstalk ratio in the telephone channels. The Comité Consultatif International Téléphonique have recommended[3] that this ratio should be at least 58 decibels.

In testing the cable, it is convenient to make all crosstalk measurements on 6-mile sections, the 0·975-inch tubes being connected through at the 3-mile points. It is thus necessary to find a relation between the values so measured and the overall signal-to-crosstalk ratio for a complete route, for specified combinations of tubes and repeater spacings. It is assumed that the route might ultimately be 600 miles long, but that there would be frequency-translating equipment, e.g. to bring the telephone channels down to the range 60–108 kilocycles, every 150 miles. Over the whole route the near-end crosstalk contributions from successive repeater sections will tend to add on a power basis, because the total lengths of cable that they traverse differ in a non-systematic way on account of the slight differences in length of the repeater sections. For far-end crosstalk the components from successive repeater sections add on a voltage basis* between the 150-mile terminal points. The contributions of the four 150-mile lengths will, however, add, or can be made to add, on a power basis because approximately random phase-changes can be introduced in the frequency-translating equipment.

Making these assumptions the expressions given in Table 1 have been derived for the minimum crosstalk attenuation that must be realised on individual 6-mile lengths, to obtain an overall signal-to-crosstalk ratio of 58 decibels for a 600-mile route.

[3] Programme Général d'Interconnexion Téléphonique en Europe (1947–1952), Comité Consultatif International Téléphonique, Montreux, 1946.

* This is strictly true only if both tubes have the same velocity of propagation; where the velocities differ, as they will for the 0·375-inch and 0·975-inch tubes, the components will not all add in phase, and the conditions will be less stringent. This advantage is neglected here.

The above expressions contain a term p to take account of any difference in repeater output level between the two tubes in any 4-kilocycle band. At 60 kilocycles, where telephony only is of interest, it is assumed that there will be no systematic differences in level and p is fixed at zero.

TABLE 1

Minimum Crosstalk Attenuation Required on 6-Mile Lengths of Cable

Conditions	Attenuation in Decibels	
	Near-End	Far-End
Between 0·375-inch tubes with 6-mile repeater spacing	$N_1 = 78 + p + 0{\cdot}72 f_k^{\frac{1}{2}}$	$F_1 = 92 + p + 0{\cdot}72 f_k^{\frac{1}{2}}$
Between 0·975-inch tubes with 12-mile repeater spacing	$N_2 = 76 + p + 0{\cdot}60 f_k^{\frac{1}{2}}$	$F_2 = 92 + p + 0{\cdot}30 f_k^{\frac{1}{2}}$
From 0·975-inch tubes with repeaters at 12-mile spacing to 0·375-inch tubes with repeaters at 6-mile spacing	$N_3 = 77 + p + 0{\cdot}72 f_k^{\frac{1}{2}}$	$F_3 = 91 + p + 0{\cdot}72 f_k^{\frac{1}{2}}$

Symbols used

f_k = Frequency in kilocycles.

N_1, N_2, N_3 = Near-end crosstalk attenuation measured on a 6-mile section.

F_1, F_2, F_3 = Far-end crosstalk attenuation measured on a 6-mile section.

p = Ratio in decibels of output in any 4-kilocycle band from repeater on an interfering tube to output from repeater on disturbed tube.

Note.—In some cases it has been necessary to make certain approximations to obtain a simple formula covering the whole frequency band. All the formulae are strictly correct at 60 kilocycles but some give crosstalk requirements erring slightly on the side of stringency at higher frequencies.

For the transmission of television, carrier levels as much as 35 decibels higher than the levels used in testing the telephone channels might conceivably be employed, which would involve an increase of 35 decibels in the crosstalk attenuation required at all possible television carrier frequencies. In the neighbourhood of 500 kilocycles this would be a stringent requirement. It should, however, be practicable to make the television carrier frequency correspond to a gap between supergroups in the telephone spectrum, so that the margin need only cover the levels corresponding to concentrations of energy in the television sidebands. This margin will be taken as 20 decibels, and the requirements at 500 kilocycles and above are, therefore, given by the expressions in the table with p equal to 20. To allow for television signal components on the cable down to 300 kilocycles, the factor p is specified as increasing uniformly with frequency from zero at 300 kilocycles to 20 decibels at 500 kilocycles.

As it is very unlikely that all the adverse conditions assumed will be encountered together no allowance has been made for crosstalk contributions from several tubes simultaneously.

2.4 Power for Repeaters

It is inconvenient and expensive to provide a separate connection to the mains at each repeater station on a coaxial cable to supply power for the repeater equipment; moreover, such an arrangement would introduce a large number of points at which the supply might fail, with a greater likelihood of interruption when power is derived from a rural network, and would involve the extensive provision of emergency supplies. It is therefore normal practice to supply power to a coaxial cable at selected points only, and transmit it over the cable itself to intermediate stations.

When the present cable was first planned it seemed likely that power for as many as five intermediate stations, at 3-mile intervals, might have to be fed over the large tubes in some sections of the cable.† Assuming that the equipment for each tube at each station might require 400 watts of power and that the conductor resistance would be approximately 7 ohms per mile, it appeared that the large tubes at a power feeding point might have to operate at about 640 root-mean-square volts. To be sure of a substantial factor of safety the large tubes are required to withstand a two-minute application of 2000 root-mean-square volts at 50 cycles.

3. *Early Experimental Work*

As soon as the principal requirements of the large television tubes were established consideration was given to the mechanical features involved in a practical cable design. To meet the impedance uniformity limits a high degree of precision is required in the dimensions of both

† Later it turned out that only four stations need be catered for.

the inner and outer copper conductors and also in regulating the quantity of solid dielectric per unit length of tube. In factory lengths it is desirable to hold the external diameter of the inner conductor to within less than 0·001 inch and the effective internal diameter of the outer conductor to within 0·0025 inch of the average values. At the top frequency for which the cable has been designed, skin effect limits the useful thickness of the inner and outer conductors to a few mils.; it is therefore desirable to economise material by making the conductors in the form of thin cylindrical tubes. The quantity of solid dielectric must also be reduced to a minimum on account of cost and to reduce dielectric loss.

Operating against the electrical requirements, however, is the fact that the tubes must be able to withstand factory handling involving reeling on to drums of a size suitable for the subsequent operations of cabling, lead covering, and protection. The completed cable must also be capable of withstanding the handling encountered during installation.

3.1 Inner Conductor

Experience with the standard 0·375 inch coaxial cable employing the interlocking-tooth type of outer conductor indicated that for the large tubes a centre conductor constructed along similar lines could be expected to provide a high degree of uniformity of diameter and to be flexible enough to meet the manufacturing and installation conditions. To keep the seam closed some external binding or clamping is necessary, but it is undesirable to provide a continuous binding of insulation over the inner tube since this would introduce an undesirable amount of dielectric in the most intense portion of the electric field. This problem has been solved by using clamping discs which form part of the insulating spacers in the tube.

To provide a smooth inner conductor, the interlocking teeth project inside the tube, which has, therefore, a smooth cylindrical external surface with a single longitudinal seam at the abutting edges.

3.2 Dielectric

Of the many known dielectrics only two are generally available having low permittivity and loss at the frequencies involved. These are polystyrene and polythene. Polystyrene, unless used in thin films or threads, tends to be less flexible than polythene and for this reason polythene is preferred. Its permittivity is 2·3 and its power factor 0·0004.

In the early experiments consideration was given to the extrusion of a continuous tube of polythene to provide support for the outer conductor. However, the difficulty of controlling concentricity and external diameter to tolerable limits ruled out this method. The final solution adopted is to use the injection moulding technique to produce short thin-walled cylinders of polythene having a disc with central hole to support the centre conductor and a spigot at one end, the spigot making joint with the next cylindrical spool in the cable. These spools are slit radially to enable them to be placed in position on the inner conductor. They can be made with a high dimensional accuracy and the method of insulation has the advantage of allowing the conductor spacing to be controlled to very close tolerances during manufacture. The spools used have a centre-to-centre spacing of the discs of 1·56 inches, a wall thickness of 0·080 inch and an external diameter of 0·940 inch.

3.3 Outer Conductor

Electrically the outer conductor also should be a smooth thin-walled cylinder. The internal diameter necessary to meet the attenuation requirements is 0·975 inch and a thin-walled tube of this diameter will buckle and crack when subjected to frequent bends such as occur during subsequent manufacturing operations. The conventional methods of making a flexible hollow tube are to produce a tube from spirally-wound tapes or to form transverse corrugations in the wall of the tube. The use of spiral tapes introduces a spiral component into the current passing along the cable and it is considered preferable to corrugate a tape and form it into a tube with a single longitudinal seam, although this involves some additional attenuation introduced by the corrugations. However, by controlling the degree of corrugation the increase in attenuation is limited and can be allowed for in the design.

In the first experimental cables the corrugated outer conductor tape was supported on the spool insulators, but impedance variations along the completed tubes showed that changes in the effective diameter of the outer conductor were excessive. It was therefore decided to make the outer tube self-supporting. This has been achieved by off-setting the corrugation at both edges of the tape so that when formed into a tube the two opposing edges of the tape are prevented from over-riding. In this way the diameter of the outer conductor tube is dependent only on the width of the tape from which it is formed, and can be controlled to very close limits.

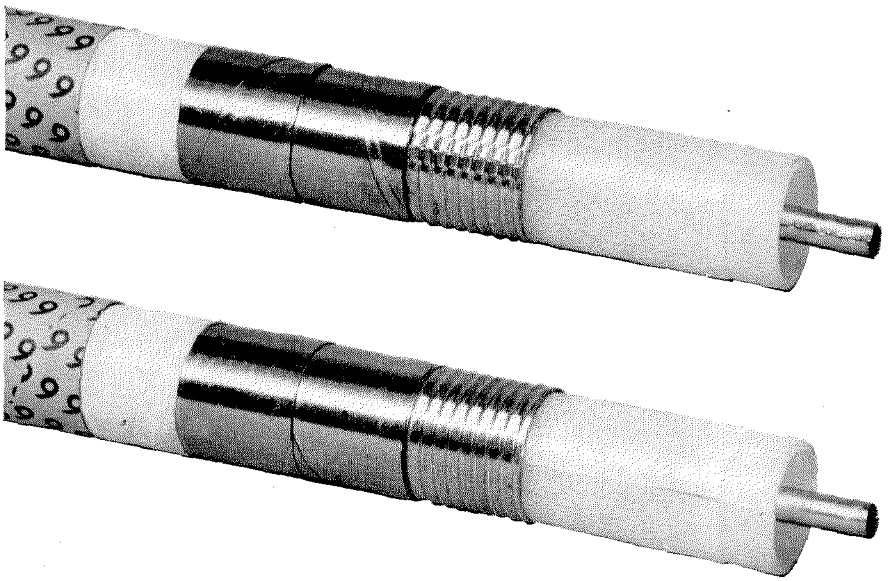


Figure 3—Views of 0·975-inch tubes.

3.4 Binding

After the outer conductor tube is formed it is necessary to hold the abutting edges together by binding the tube; the binding must also provide mechanical strength and add to the crosstalk attenuation between the tube and other tubes and pairs in the cable. The requirements are met by providing a binding of four steel tapes each 5 mils thick, the first pair of tapes being applied with right-hand lay and the second pair with left-hand lay. In this way, in addition to meeting the above requirements, torsional rigidity is imparted to the tube, which enables it to withstand subsequent handling.

To retain the steel tape binding under the appropriate tension when the tubes are cut for testing and jointing purposes and to insulate the outer conductor for testing, a binding of adhesive insulating tape is applied directly over the steel tapes and a final lapping of numbered paper is provided for identification.

4. *Final Cable Design*

The above description outlines the main channel of development that led to the final cable design, although many subsidiary lines were explored before various alternatives were discarded. The final design of the large tubes is therefore as follows:—

Centre Conductor: Hollow tube constructed of 0·010-inch thick copper tape with internally interlocking teeth.
External diameter of tube: 0·250 inch.

Dielectric: Polythene spools having an effective length of 1·55 inches and an external diameter of 0·940 inch.

Outer Conductor: Transversely corrugated tube formed from 0·010-inch-thick copper tape with offset corrugation on the edges.
Effective internal diameter: 0·975 inch.

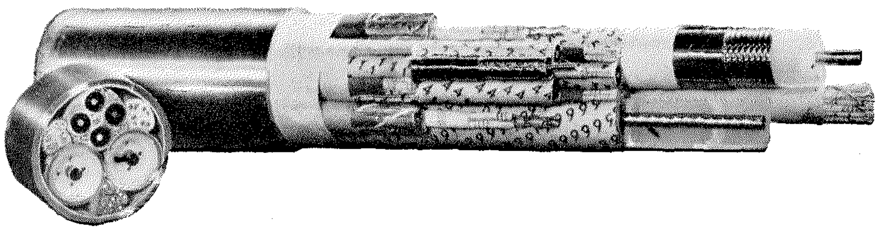


Figure 4—Lay-up of composite cable.

Binding: Two 0·005-inch steel tapes breaking joint with right-hand lay. Two 0·005-inch steel tapes breaking joint with left-hand lay.
One adhesive insulating tape.
One numbered paper tape.
External diameter: 1·080 inches.

Figure 3 shows two views of the large tubes. In the upper view the abutting edges of the outer conductor are visible and in the lower one the tube is turned round to show the radial slit in the polythene spool.

The 0·375-inch tubes and balanced pairs in the cable conform to existing practice and call for no comment. The lay-up of the complete cable has already been described in Section 1.3, following a discussion of the factors determining the diameter of the large tubes. Figure 4 shows specimens of the cable as laid. (In the specimens the protective covering of bitumen and hessian with which the cable is served overall has been removed.)

4.1 Joints

In arriving at a design of joint for connecting together factory lengths of cable during installation on the cable route, the following principal requirements have to be met:—

A. The joint must not cause appreciable reflections of the energy transmitted through it in the frequency spectrum 1 to 26 megacycles.

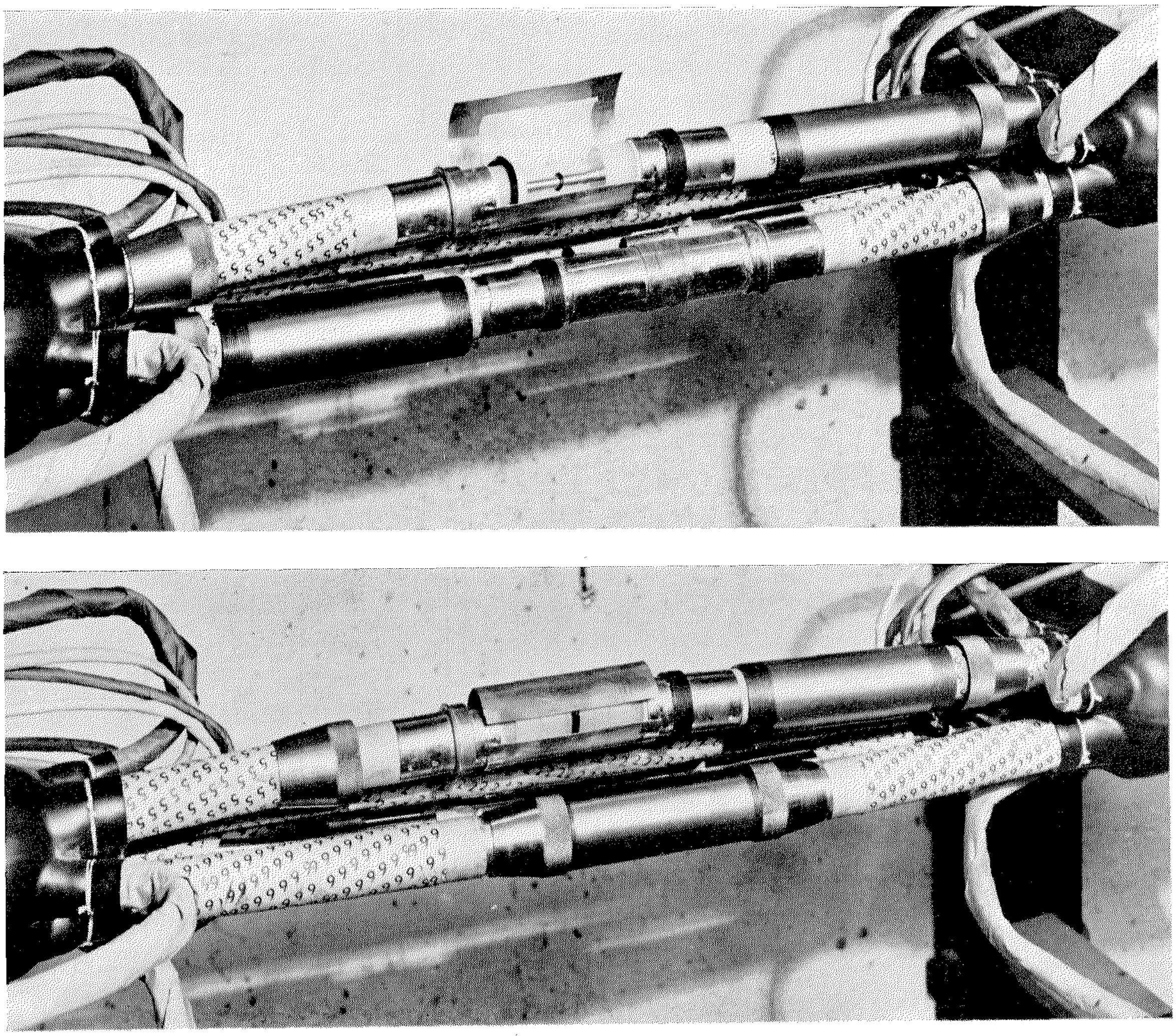


Figure 5—Construction of joints in 0·975-inch tubes.

B. A water barrier must be provided in accordance with Post Office practice for all air-spaced telephone cables.

C. The joint must be so constructed that during assembly, and when, if the occasion should arise, the joint is opened, there should be no risk of damaging the dielectric structure of the cores or of the joint itself, by melting of the polythene insulators as a result of the soldering operations on the conductors.

For the first requirement it was found that a symmetrical disposition of the components on either side of a solid polythene disc, forming with two rubber rings the required water barrier, could be so proportioned that although elemental portions differed in characteristic impedance from the cores and therefore caused reflections of the transmitted wave, the vector resultant of all the reflections occurring in the joint was sensibly zero over the whole frequency spectrum up to 26 megacycles or more. Two views of the joint finally adopted are shown in Figure 5; the joint is described below:—

A simple procedure has been evolved for cutting the ends of the two cores to be jointed so that the distances to the first polythene discs on either side of the joint are the same, and are between 0·5 and 1·0 times one polythene spacer length. The centre tubes of the core are connected together by means of a copper rod inserted into the open ends and soldered. Over this rod the split polythene spacer with central disc, previously mentioned, is tied firmly into position over a small rubber ring. During the soldering operations required to fix a steel collar and copper ring to each outer conductor the polythene spacers are temporarily removed from the core end for a sufficient distance to avoid any distortion of spacers due to excessive heating. After replacing these spacers the joint between the two outer conductors is made by means of a cylindrically formed thin copper plate which is seam-soldered at either end and along the longitudinal seam. Since this soldering operation does not involve heating large masses of metal there is no melting of the polythene spacers inside the sleeve. To improve the engagement between the inner surface of the outer copper sleeve and the rubber ring encircling the outside of the split polythene joint spacer, a tight binding of copper wire extending for approximately 1 inch is applied to the centre of the sleeve and soldered into position. To complete the screening of the joint and to render its mechanical strength independent of the outer copper sleeve, a steel sleeve with locking rings is screwed to steel collars previously soldered to the steel tapes and outer conductors of the cores. The joint, which is finally wrapped in empire cloth, is approximately 7 inches long and $1\frac{1}{2}$ inches in diameter. In the complete cable joint there are two of these joints for the 0·975-inch tubes, four smaller joints for the 0·375-inch tubes, and the usual joints in the unscreened and screened pairs, the overall dimensions being 30 inches long by 4 inches in diameter. To prevent passage of water along the interstices between the various coaxial tubes and the pairs in the main joint a pair of multi-outlet plastic sleeves is fitted tightly over all the components where they leave the lead sheath of the cable inside the outer lead sleeve of the joint.

4.2 Experimental Cable

When the preliminary development work had resulted in the production of a number of drum lengths, it was decided to manufacture and joint together in the factory a substantial length of cable to allow the repeater-section characteristics of the large tubes to be determined; in particular to check the effect of the joints on impedance uniformity. This would also enable measurements of attenuation, impedance uniformity, and crosstalk to be made, which could not be measured accurately on individual lengths. Upon the completion of this work the measurements, which were made on 2·3 miles of cable, comprising 33 drum lengths, showed that the attenuation and crosstalk were satisfactory, but that the variation of impedance of the large tubes was rather more than had been expected. A detailed examination of the test results on individual drum lengths and a comparison with the tests on the 33 lengths in tandem showed that the dimensional accuracy of the outer tube was insufficient to meet the high degree of uniformity required. The construction of the large tubes was then altered somewhat, as indicated previously, the principal changes being the use of a self-supporting outer conductor, the shortening of the spacing spools, and improvements in the outer binding.

A mile of cable of the modified design was manufactured, jointed in the factory, and tested, and the results obtained showed that the requisite uniformity of characteristic impedance had been achieved. Manufacture of a full repeater section of cable was therefore put in hand. Meanwhile the mile of cable was installed on the London-Birmingham route and retested. The results after installation showed virtually no change from the factory results, thus indicating that the cable had the necessary degree of stability to withstand handling during installation.

The repeater section was manufactured and installed on the London-Birmingham route between Gladstone and Cunningham repeater stations. This repeater section was then completely tested and satisfied all requirements.

4.3 Details of Cable Route

A line diagram of the main and tail cable routes is shown in Figure 6. The cable from Alexandra Palace to Birmingham has the lay-up already illustrated in Figures 2 and 4, but between Birmingham and the transmitter at Sutton Coldfield the four 0·375-inch tubes and associated balanced pairs are omitted, because one-way television, only, need be catered for on this part of the route. The places at which the three-mile repeaters will be located, and the power feeding points, are shown in the line diagram.

4.4 Cable Terminations

The method adopted for terminating the cable, as arranged at an intermediate station, is shown in Figure 7. The cable uprising against the wall is separated into short lengths of individual lead-sheathed coaxial tubes and groups of balanced pairs at the plug joint. The coaxial tubes are carried over racking to the point where they finish in sealing ends designed to provide an airtight termination to which flexible cable can be connected through a plug and socket without

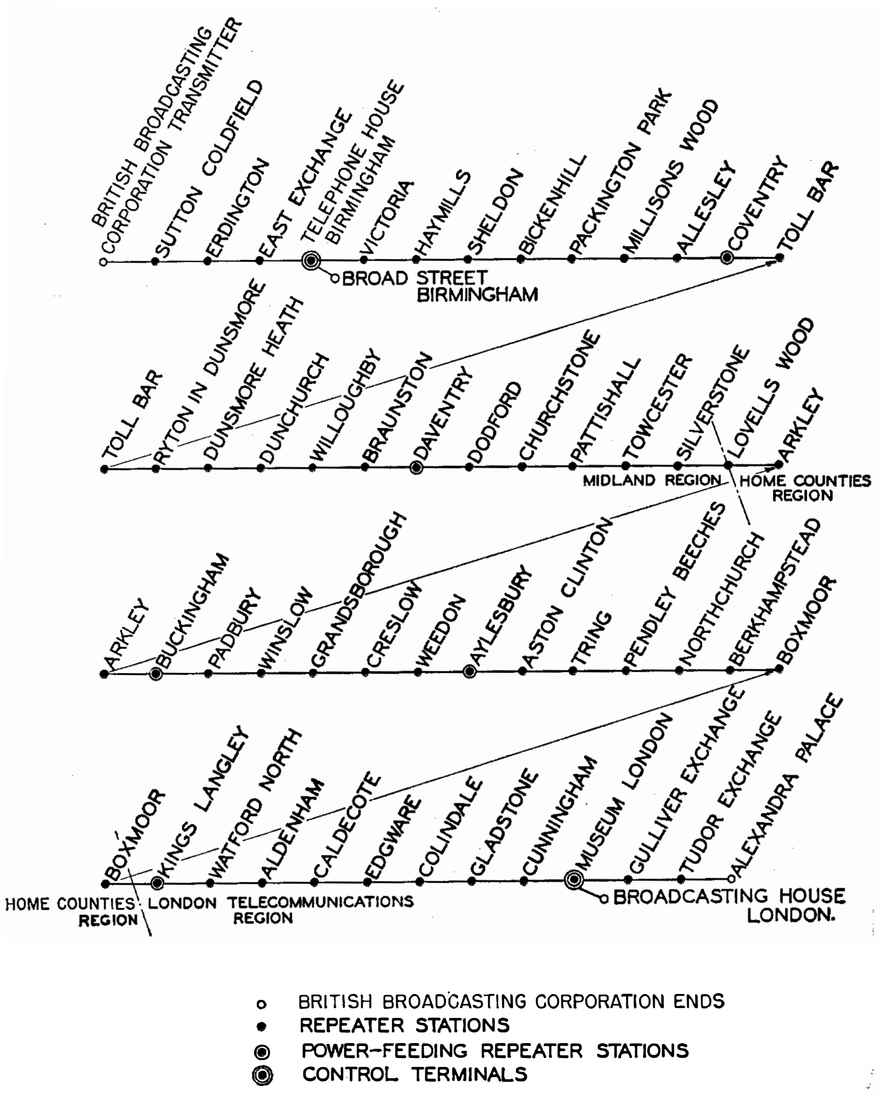


Figure 6—Line diagram of cable routes.

introducing an impedance irregularity. Each sealing end provides an earth for the outer conductor which can be removed for testing purposes.

It will be seen that in this particular station all the coaxial tubes terminate in sealing ends in this way, and that the balanced pairs also are terminated in test tablets, shown on the right-hand side of the picture. But sealing ends for the 0·375-inch tubes and test tablets for the balanced pairs are provided only at the 6- and 24-mile repeater stations, and at intermediate points permanent through connections are provided inside plumbed joints.

Figure 7—Cable terminations.

5. Test Results

Although the cable has not yet been completely installed it is possible to give some results that are typical of the sections already tested:—

Characteristic Impedance: Figure 8 shows the way in which the resistive component of the input impedance of a 0·975-inch tube varies with frequency at one end of a three-mile repeater section, when the other end is terminated in 75 ohms. The sum of the six largest differences of impedance corresponding to this curve is 22·1 ohms, compared with the specification figure of 26·5 ohms.

In Figure 9 a copy is shown of the complex echo pattern observed on a cathode-ray oscilloscope at the sending end of a three-mile section, when a 0·1 micro-second direct-current pulse is applied. This type of test is very useful for locating irregularities and diagnosing them. The break-through of the sent pulse, and an echo from the far-end terminating impedance, appear at the beginning and end of the time scale respectively and it will be seen that the biggest echo returned from within the section is 60 decibels below the

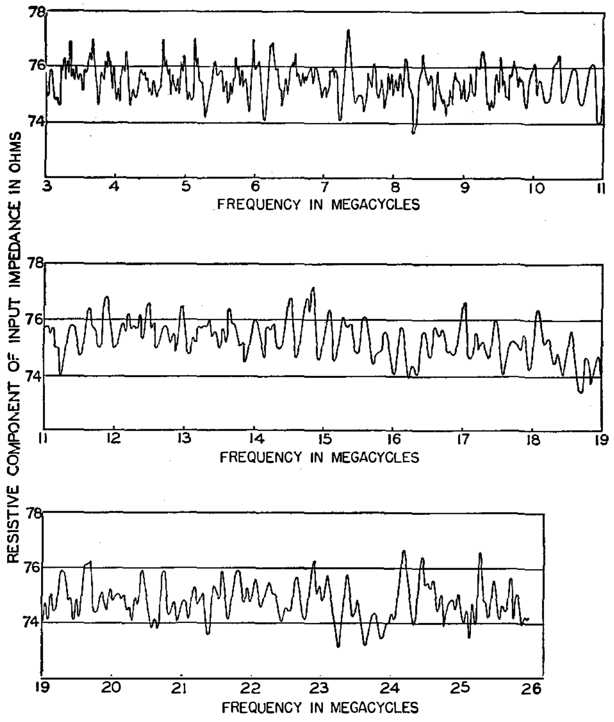


Figure 8—Variation of resistive component of input impedance with frequency for 3-mile length of 0·975-inch tube.

sent pulse and arises 740 yards from the sending end. The pair of echoes of opposite sign, arising 1900 and 2050 yards from the sending end, suggests that on one drum length the characteristic impedance of the tube tested is slightly lower than that of adjacent lengths. Since an echo near the sending end 60 decibels below the level of the sent pulse, corresponds to a change of impedance of the order of 0·15 ohm it will be seen that the response shown in Figure 9 indicates a very high degree of impedance uniformity.

On all repeater-section ends so far tested the mean characteristic impedance around 20 megacycles lies between 74·8 and 75·2 ohms, which should be compared with the specification limits of 74 and 76 ohms.

Attenuation: There is considerable difficulty, having made accurate measurements of cable attenuation, in correcting them to a reference temperature, because of the difficulty of determining the temperature of the cable, which can vary along its length. However, it appears from precise measurements on a section of the cable that at 15 degrees centigrade the attenuation of

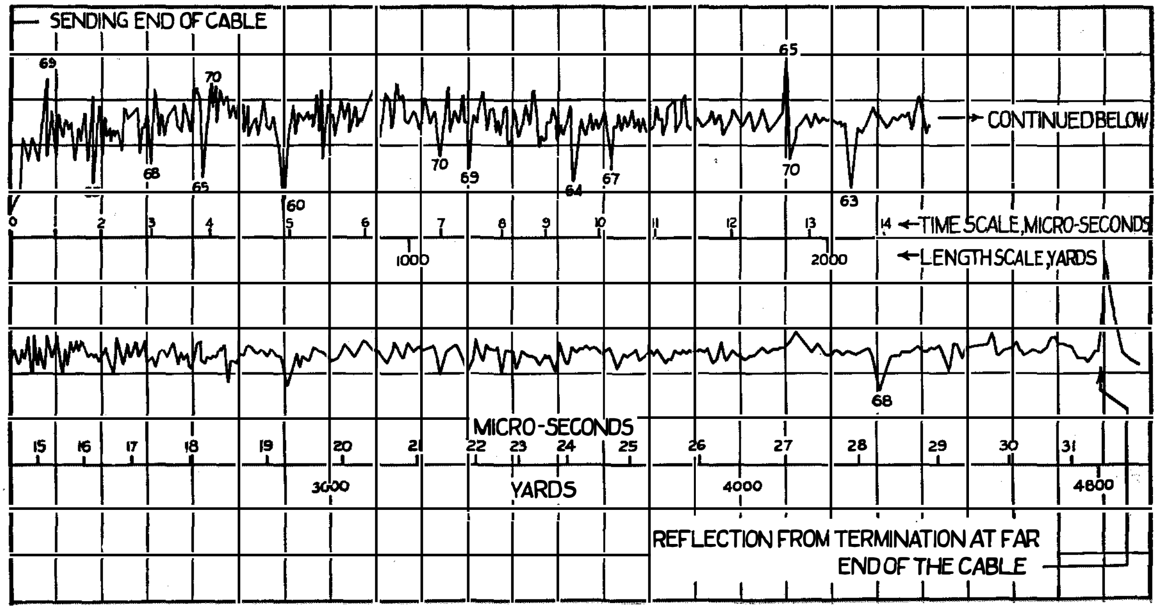
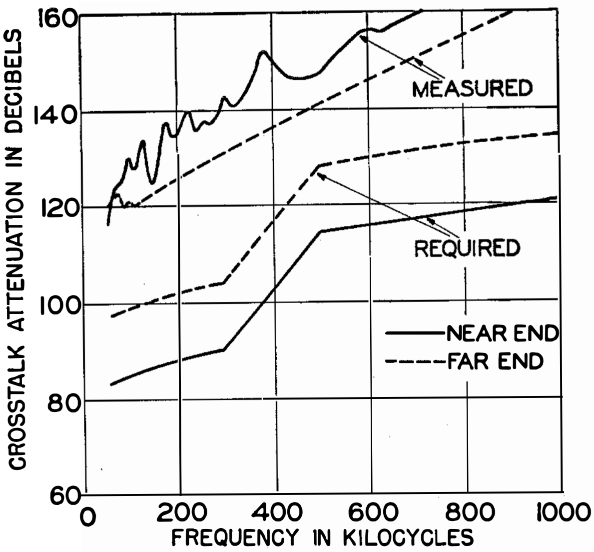
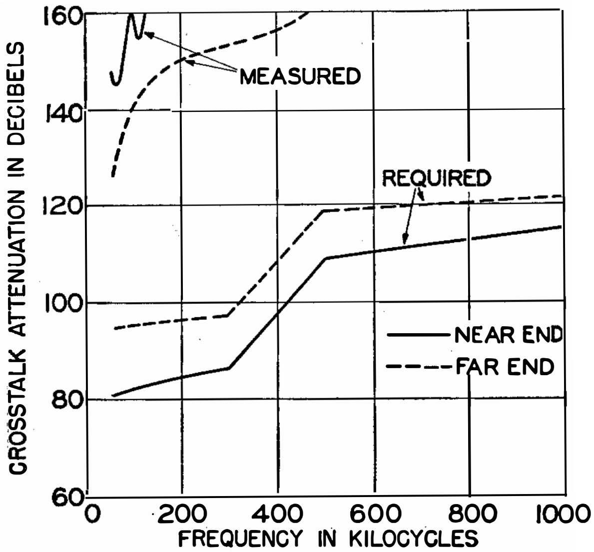


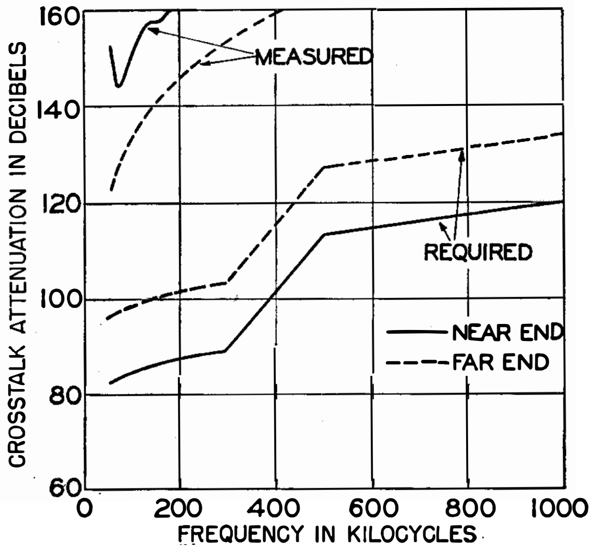
Figure 9—Pulse test on 2·73-mile section of 0·975-inch tube. Approximate rectangular direct-current pulse of 0·1 microsecond width. Figures appended to principal reflections give their magnitude expressed in decibels below incident pulse level.



Between 0·375-inch tubes.



Between 0·975-inch tubes.



Between one 0·375-inch and one 0·975-inch tube.

Figure 10—Crosstalk attenuations for 6-mile sections.

the 0·975-inch tubes conforms to the law:—

$$A = 1{\cdot}55\, f^{\frac{1}{2}} + 0{\cdot}011\, f \text{ decibels per mile,}$$

where f is the frequency in megacycles per second. The maximum departure from the attenuation law as specified in Section 2.2 occurs at 1 megacycle and amounts to $-0{\cdot}65$ per cent. The change of attenuation that occurs when the cable is drawn into the duct appears to be less than 0·3 per cent.

Crosstalk. Typical crosstalk attenuation results are shown in Figure 10 with the minimum requirements indicated for comparison. It will be seen that on far-end crosstalk between 0·375-inch tubes there is a margin of at least 10 decibels and that in all other cases the requirements are satisfied with much greater margins.

6. *Acknowledgments*

The authors have much pleasure in acknowledging their indebtedness to their colleagues in the Post Office and in Standard Telephones and Cables, Limited, who have been associated with them in various aspects of this work, in particular to Messrs. E. C. H. Seaman and E. Baguley who have also assisted them in the preparation of this article.

Highly Balanced Radio-Frequency Transmission Lines

By K. H. ZIMMERMANN

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WHEN it is desired to transmit power from a balanced, push-pull, or symmetrical source, neither side of which is grounded, the balanced twinax line is preferable to a coaxial line.

In appearance, the twinax line shown in Figure 1 resembles the familiar coaxial line, except that there are two inner conductors instead of a single centrally disposed conductor.

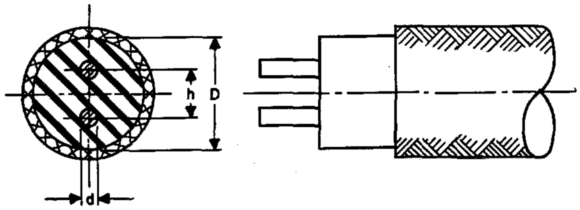


Figure 1—Construction of twinax balanced shielded transmission line.

The position of the two inner conductors with relation to each other and to the surrounding shield is extremely important in determining the electrical characteristics of the line.

One of the most important properties of a high-frequency line is its characteristic impedance. Design formulas for twinax lines are more complicated than those for coaxial lines. In addition, the formulas are not as accurate, and it is usually necessary to modify the first design to obtain the desired results. The characteristic impedance of a two-conductor shielded line is obtained from the following relation:

$$Z_0=\frac{276}{(k)^{\frac{1}{2}}}\log_{10}\left(2v\frac{1-Q^2}{1+Q^2}\right)-120\left(\frac{1+4v^2}{16\,v^4}\right)(1-4Q^2), \quad (1)$$

where Z_0 = characteristic impedance.

d = diameter of inner conductors.

k = dielectric constant of insulating material.

h = spacing between inner conductors, center to center.

D = diameter of dielectric.

$Q=h/D$.

$v=h/d$.

The importance of conductor spacing is shown in the curves of Figures 2 and 3. These curves were prepared for two different lines. Figure 3 also shows the effect of variation in the diameter of the dielectric. Although there is considerable difference in physical size, both lines have a characteristic impedance of 95 ohms.

The first balanced lines consisted of two parallel conductors, equally spaced from the center of the dielectric. A braided copper shield is applied over the dielectric, and a thermoplastic jacket extruded over the braid serves as a protective covering.

These lines are still in demand and serve effectively in many applications. However, the parallel-conductor line is flexible only in the direction at right angles to the plane through the two conductors. This difficulty has been eliminated by separately insulating each inner conductor to a diameter equal to the desired center-to-center conductor spacing. The insulated conductors are then twisted together and another extrusion of dielectric is applied to obtain the final circular cross section. This not only increases the flexibility of the line but also improves the

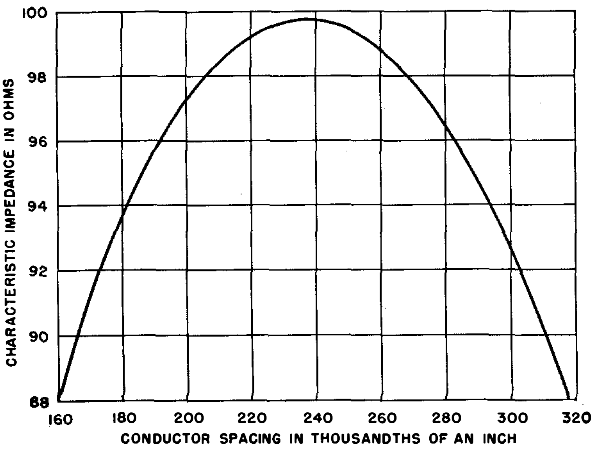


Figure 2—Calculated values of characteristic impedance plotted against conductor spacing for *K-56*-type twinax cable. Diameter of the dielectric is 0.475 inch and dielectric constant is 2.34. Each conductor consists of 7/0.0285-inch copper wire.

electrical balance by providing a closer tolerance on the conductor spacing. Due to the twisting, the conductors change their positions relative to the shield and there is less tendency toward unbalance than in the parallel arrangement, where it is possible for one conductor to be closer to the shield for the entire length of the line.

Balance ratio of a transmission line is defined as the vector sum over the vector difference of the voltages across the two inner conductors at the receiving end of the line when voltage at the transmitting end is applied between the two inner conductors tied together and the shield. Theoretically, for an ideal line, the balance ratio is infinite.

To obtain a highly balanced line, it is necessary to exercise meticulous care during each manufacturing process. First, each inner conductor is insulated to a dielectric diameter that is held to ±0.002 inch. It is important that each insulated conductor be identical in all electrical and physical characteristics. Both conductors are obtained from the same manufacturing run to insure identical characteristics.

Twisting the inner conductors is another critical operation. If the conductors are not maintained at equal tensions during twisting, one conductor will twist about the other and will become the longer of the two in the completed assembly. Consequently, all tensions at the twister take-off reels must be carefully maintained.

Usually, when conductors are twisted, a filler material is inserted in the interstices or valleys to obtain a round cross section. This is not possible in twinax construction because a homogeneous dielectric, excluding all foreign materials, is required. Therefore, another layer of the same dielectric material is extruded over the inner conductors. Success in this difficult operation depends on the design of the extrusion tools and the skill of the operators. Any variations in centering or failure to meet dimensional tolerances will result in lines that are not suitable in appearance, characteristic impedance, or balance ratio.

The braided shield and thermoplastic jacket are applied exactly as in the manufacture of the familiar coaxial line.

When the line is complete, physical dimensions, dielectric strength, velocity of propagation, capacitance, capacitance unbalance, characteristic impedance, attenuation, and balance ratio are measured.

1. *Velocity of Propagation*

The velocity of propagation is obtained from the resonant frequency of a known length of line. A cable length of 1.5 meters, which is three-quarters of a wavelength at approximately 100 megacycles per second, is used. The resonant frequency of the 1.5-meter length is found by

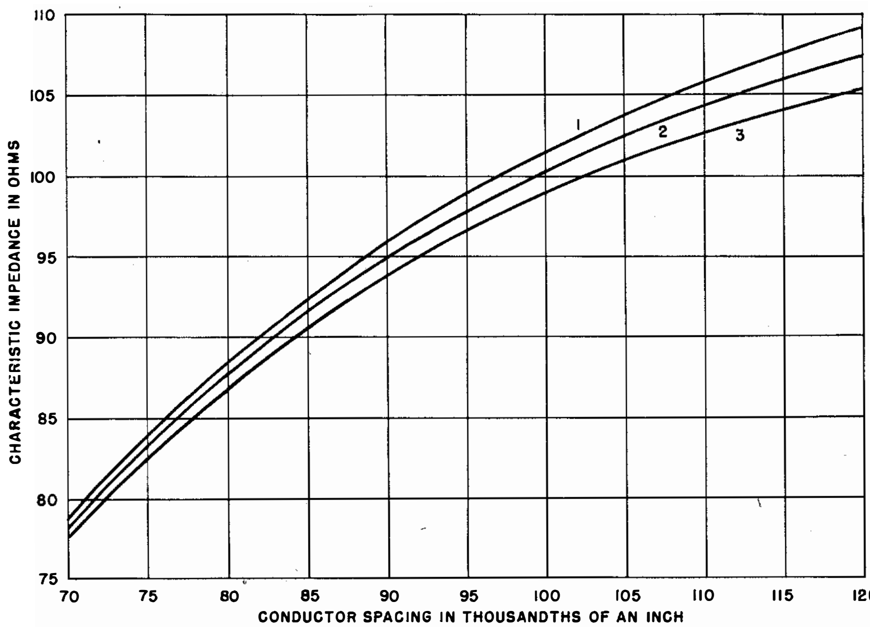


Figure 3—Calculated values of characteristic impedance plotted against conductor spacing for *K-51*-type twinax cable. The dielectric constant is 2.33 and the dielectric diameters differ for each of the three curves, as follows: Curve 1—0.295; Curve 2—0.285, and Curve 3—0.275 inch. The conductors are 7/0.0152-inch copper wire.

loosely coupling the line to an oscillator that is varied in frequency until resonance is indicated by a dip in a grid meter. For a 1.5-meter length of line, the velocity of propagation in percent of the velocity in free space is given by the relation:

$$V = 2/3\, f, \tag{2}$$

where f is the resonant frequency in megacycles.

The velocity of propagation of a polyethylene-insulated cable is approximately 66.3 percent of the speed of light.

2. Capacitance

Three measurements are required to find the capacitance of a twinax line.

First, the capacitance C_A between one conductor and the shield is measured with the other conductor tied to the shield; second, the capacitance C_B between the second conductor and shield is measured with the first conductor tied to the shield; third, the capacitance C_C is measured between the two conductors tied together and the shield. The capacitance of the line is given by

$$C = \frac{2(C_A + C_B) - C_C}{4}. \tag{3}$$

3. Characteristic Impedance

If C is the capacitance of the 1.5-meter length of line, the characteristic impedance of the line is obtained from this value and the velocity of propagation as shown:

$$Z_0 = \frac{600{,}000}{V(\text{percent}) \times C}. \tag{4}$$

4. Balance Ratio

Two operations are required to determine the balance ratio. The line is connected between a suitable generator and a receiver having an input impedance that matches the impedance of the line. A voltage V_1 is applied between the two conductors at the generator end and the output at the receiver is noted. A voltage is then applied between the two conductors tied together at the generator end and the shield. This voltage is increased to a value V_2 at which the receiver voltage is again the same. The ratio of V_2/V_1 is the balance ratio.

Three twinax balanced lines are available commercially; one has a characteristic impedance of 72 ohms and the other two are of 95 ohms. The 72-ohm line, designated *KT-107*, was designed for use with a dipole antenna. This line matches the 72-ohm antenna and, of course, is shielded.

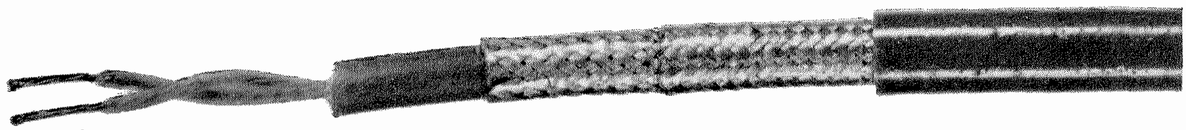
The two 95-ohm lines, *KT-51* and *KT-56*, were developed for radar applications. The larger line *KT-56* is used where the power-handling ability of the smaller line is inadequate. The properties of these lines are given in Table 1.

Balanced transmission lines are used with dipole antennas, balanced loops, or wherever it is desirable to connect to a balanced load. Measuring circuits often require a balanced line.

TABLE 1

PROPERTIES OF TWINAX LINES

Type	Characteristic Impedance in Ohms	Capacitance in Micro-microfarads Per Foot	Attenuation in Decibels Per 100 Feet at 400 Megacycles	Power Rating in Watts at 100 Megacycles	Dimensions of Diameters in Inches				Weight in Pounds Per 1000 Feet
					Conductors	Dielectric	Shield	Jacket	
KT-51	95	16	10.5	—	0.046	0.285	0.355	0.430	132
KT-56	95	16	8.8	1250	0.086	0.472	0.540	0.640	233
KT-107	72	21	10.0	—	0.036	0.146	0.191	0.250	41



KT-51 highly balanced radio-frequency transmission line.

Transmission-Measuring Set for Low-Frequency Carrier Systems

By J. BRUNDAGE and J. ZYDA

Federal Telephone and Radio Corporation, Clifton, New Jersey

IN THE 902-A transmission-measuring set, a stabilized resistance-capacitance oscillator covers the range from 300 to 40,000 cycles per second. A 4-position rotary switch provides for sending, receiving, measuring, and calibration. Transmission power may be varied from −40 to +20 decibels referred to 1 milliwatt over the frequency range with adequate sensitivity in the receiver to utilize these powers. Self-calibration is provided. The unit may be readily converted for either portable use or rack mounting.

• • •

Many industries find it necessary to maintain private telephone systems as an adjunct to their main activities. Fast dependable telephone communication is particularly important to railroads, pipe-line operators, and power companies and in some cases wire lines already in use for other services can be adapted to low-frequency carrier telephone operation. Usually, such carrier equipment is owned and maintained by the company using it.

The successful operation and maintenance of carrier telephone equipment is based on the proper adjustment of the terminal equipments to make most effective use of the characteristics of the lines, which characteristics may vary substantially with weather. As a consequence, transmission-measuring sets are used to obtain the basic information on which the adjustments are made.

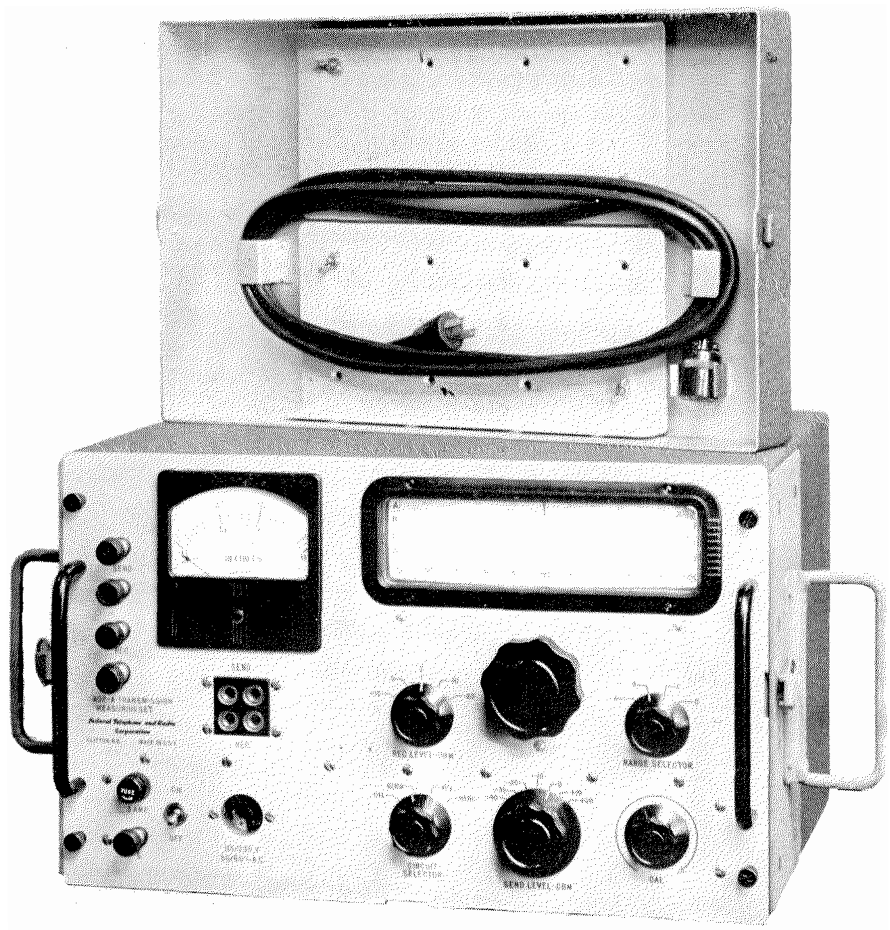


Figure 1—Transmission-measuring set arranged for portable use with cover removed. The brackets to convert to rack mounting are fastened into the cover.

Figure 2—Rack mounting of set. The handles used for portable operation may be fastened into the cover, which is not used in this arrangement.

1. Physical Characteristics

As it is desirable to use the fewest number of different types of equipment throughout a system, the 902-A transmission-measuring set was designed for either portable use or rack mounting. In Figure 1, the set is arranged for field service. The metallic case in which it is mounted has been equipped with handles. The complete unit with cover in place is $16\frac{7}{8}$ inches wide, $10\frac{1}{2}$ inches high, and $12\frac{1}{8}$ inches deep. It weighs approximately 40 pounds. The same unit may be mounted on a standard 19-inch relay rack, as shown in Figure 2, by replacing the handles with brackets. Either the handles or the brackets may be fastened in the cover when not in use.

To fulfill the field requirements of a portable instrument, the unit is entirely self-contained and includes a power supply that operates from alternating-current mains. All magnetic components and capacitors are hermetically sealed and the metallic carrying case provides further protection against weather.

2. Electrical Characteristics

A measuring set for carrier systems operating below 40 kilocycles should permit measurements at frequencies from 300 to 40,000 cycles. For field use, it should include sending, receiving and measuring circuits, a means for calibration, and the necessary power supply.

The sending circuit should provide means for transmitting any desired level from -40 to $+20$ decibels (reference level of 1 milliwatt) over the entire frequency range. It should have a 600-ohm output impedance, a frequency setting calibrated to an accuracy within 2 percent, and the wave form of the output signal should have a harmonic content of less than 3 percent.

The receiving circuit should have a sensitivity to cover the above range of levels and frequencies. The accuracy of the equipment should be within 1 decibel over the entire frequency range. Facilities must be provided to enable 600-ohm termination as well as bridging-type measurements to be made with the receiving circuit. The accuracy must not be affected by normal power-line voltage fluctuations.

Figure 3 is a block schematic showing the method used in the 902-A set to meet the above requirements.

2.1 Sending Circuit

To obtain good stability as well as to provide a convenient means for obtaining the frequency range, a stabilized circuit using a resistance-capacitance oscillator was chosen. The sending circuit shown in Figure 4 consists of a three-tube amplifier, employing a resistance-capacitance feedback network to establish and control the frequency of the oscillations. The oscillator frequency is determined by the values of resistance and capacitance in the feedback network, under control of the range-selector switch and the main tuning dial. The frequency ranges are as follows: 300 to 1200 cycles; 1000 to 4000 cycles; 3000 to 12,000 cycles; and 10,000 to 40,000 cycles. It can be shown that oscillations tend to take place at a frequency $f = \frac{1}{2}\pi RC$, where R and C are the values of resistance and capacitance used.

In Figure 4, a portion of the amplifier output is fed back through the phase-shifting network, *R1, R2, C1, C2,* to the 6G6G cathode-follower. The function of the cathode-follower is to match the necessarily high impedance of the resistance-capacitance network to the relatively low impedance of the impedance bridge, and to minimize the tube capacitance across the shunt arm of the phase-shifting network.

The bridge consists of two fixed resistors as ratio arms, a variable resistor *R3*, and a tungsten-filament lamp. The variable resistor is adjusted at the factory to give the proper output level at the SEND terminals. This occurs approximately at balance. If the signal at the grid of the second tube tends to increase, more current is drawn through the lamp increasing its resistance. An increase in lamp resistance tends to balance the bridge, which will reduce the signal applied to the second tube and, therefore, the amount of voltage fed back through the phase-shifting network. A tendency for the signal at the grid of the second tube to decrease will decrease the current through the lamp, reduce its resistance and further unbalance the bridge, thus applying a larger signal to the grid of the second tube. The over-all effect of these actions is to keep the output voltage of the resistance-coupled amplifier constant.

The output of the two *6SJ7* voltage-amplifier stages is applied through a gain control to a single-stage buffer amplifier. This is a conventional transformer-coupled power-output pentode stage using both current and voltage feedback to stabilize gain and output impedance and to reduce harmonic distortion.

A pad is connected to the output of the buffer amplifier and helps to keep the impedance constant over the frequency range. The SEND LEVEL attenuator is used in conjunction with the gain control to set the output level; the gain control being used to adjust the buffer-amplifier output level to a standard value and the attenuator to control the output level to the device under test in 10-decibel steps from +20 to −40 decibels. The signal is applied to the SEND terminals and jacks through a shielded repeating coil thus enabling the set to be used on either grounded or ungrounded lines or equipment. The connections between the jacks and terminals are so arranged that plugging into the jacks disconnects the terminals, thus eliminating the possibility of errors due to leaving circuits connected to the

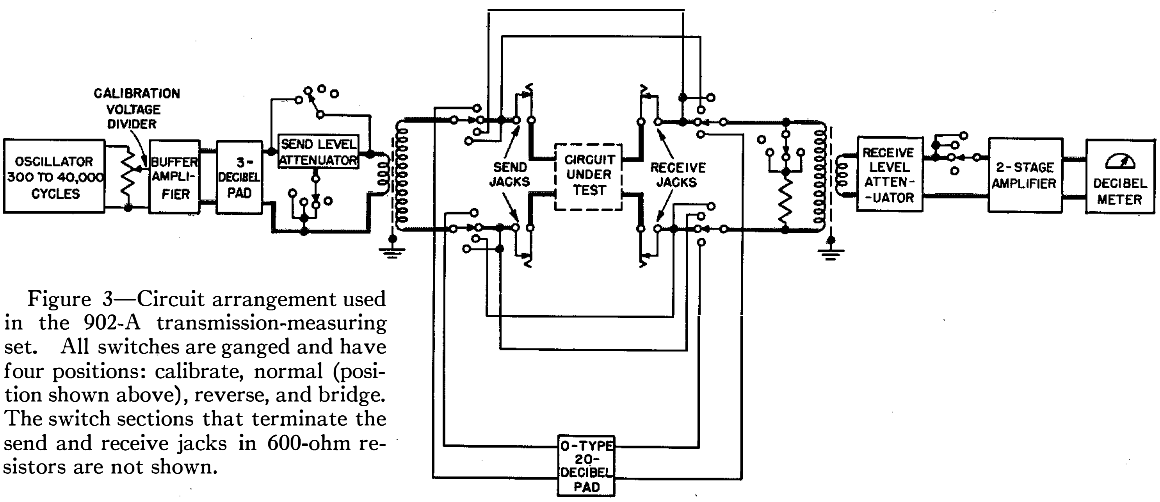


Figure 3—Circuit arrangement used in the 902-A transmission-measuring set. All switches are ganged and have four positions: calibrate, normal (position shown above), reverse, and bridge. The switch sections that terminate the send and receive jacks in 600-ohm resistors are not shown.

terminals while using the jacks. The output impedance looking back into the SEND jacks or terminals is 600 ohms.

2.2 Receiving Circuit

The receiving circuit is designed for either bridging or terminating measurements and consists of an input circuit, amplifier, and meter circuit. The input circuit is connected to the RECEIVE terminals and jacks, which are interconnected in a manner similar to the SEND terminals and jacks. Incoming signals are applied to the amplifier through a shielded repeating coil, enabling the set to be used on grounded or ungrounded circuits; and through the RECEIVE LEVEL attenuator, which permits the signal on the grid of the first amplifier tube to be set at the proper value. The nominal input impedance of the circuit is 5000 ohms and this is shunted to 600 ohms on the NORMAL and REVERSE positions of the CIRCUIT SELECTOR SWITCH.

The amplifier is a conventional two-stage resistance-coupled amplifier with adjustable negative current feedback over both stages to stabilize gain. The feedback control is adjusted and locked at the factory to standardize the amplifier gain.

2.3 Measuring Circuit

The rectifier-type meter in the feedback circuit of the amplifier is calibrated in terms of the net power input level in decibels above 1 milliwatt in the 600-ohm load. It is marked with two linear scales in $\frac{1}{2}$-decibel intervals. One scale with red numerals is calibrated from 0 to 15 with zero at the left, and is used in conjunction with red 0 and red +10 range positions of the RECEIVE LEVEL control to measure levels from 0 to +25 decibels. A scale with black numerals from 0 to 15, with zero at the right, is used in conjunction with black 0, −10, and −20 range positions of the RECEIVE LEVEL control to measure levels from 0 to −35 decibels. The levels that can be measured are from −35 to +25 decibels.

Switching operations are performed by a twelve-circuit four-position rotary-type switch mounted on the front panel. In the calibration position, the sending circuit is connected directly to the receiving circuit through an accurate 20-

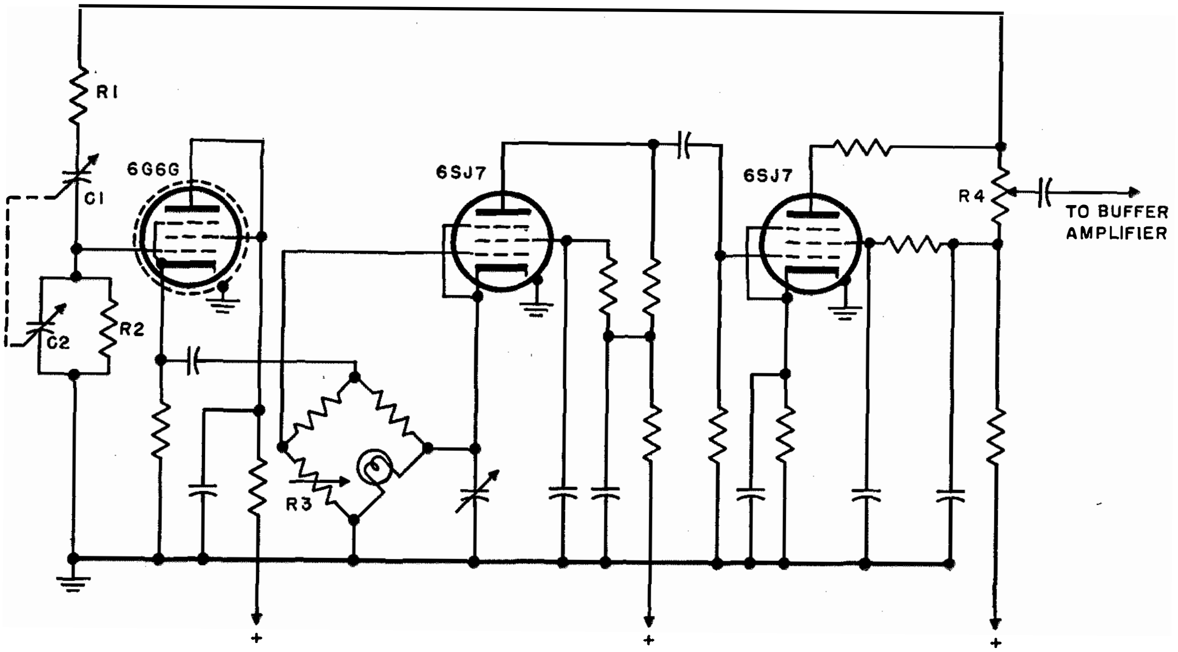


Figure 4—Resistance-capacitance oscillator for generating frequencies between 300 and 40,000 cycles. The *6G6G* is operated as a cathode-follower and a bridge circuit using a tungsten-filament lamp as one arm stabilizes the alternating voltage applied to the grid of the first *6SJ7* amplifier tube.

decibel pad. Other sections of the switch remove the SEND LEVEL attenuator from the circuit, apply 600-ohm resistors to terminate the SEND and RECEIVE jacks, and connect the receiving circuit to the 0-decibel tap on the RECEIVE LEVEL attenuator. Adjustment of the calibration voltage divider *R4* to bring the meter indication to 0 decibels standardizes the level fed into the SEND LEVEL attenuator at +20 decibels.

In the normal position of the circuit selector switch, the oscillator output is connected to the SEND jacks through the SEND LEVEL attenuator. The RECEIVE jacks are connected to the input of the receiving circuit. The unit is then ready for measurements on a 600-ohm basis.

In the reverse position of the circuit selector switch, the functions of the SEND and RECEIVE jacks are interchanged, i.e., the oscillator output is applied to the RECEIVE jacks and the receiving circuit to the SEND jacks. This enables measurements to be made around a loop in a reverse direction by merely operating the switch.

In the bridging position, the oscillator circuit is connected to the SEND jacks in the same way as in the normal position. The receiving circuit is connected to the RECEIVE jacks but with the terminating resistor removed. This makes it possible to bridge on a 600-ohm line and measure levels directly. The input impedance on the bridging position is approximately 5000 ohms.

2.4 Power Supply

The power supply built into the unit operates from either a 115-volt ±10 percent or 230-volt ±10 percent 50/60-cycle 75-watt alternating-current source (depending on the strapping of the power transformer) to provide all voltages necessary for the operation of the equipment. The alternating voltages for the tube heaters are supplied by a center-tapped winding on the power transformer. The plate and screen direct voltages are derived from a conventional full-wave rectifier circuit and associated filter networks. Part of the rectifier output passes through a resistance-capacitance filter to the receiving circuit, while the other part goes through a pi-section inductance-capacitance filter to a voltage-regulating circuit and thence to the sending circuit. Employment of separate filters for the sending and receiving circuits effectively decouples these two circuits from each other and prevents interference between them.

Recent Telecommunication Development

Organization of Telecommunications of Greece

THE GOVERNMENT of the Kingdom of Greece has entered into a technical advisory contract with the International Telephone and Telegraph Corporation for the reorganization of the telecommunications system of that country. The program is being sponsored by the Economic Cooperation Administration in Greece.

A staff of communication experts will be provided by the International Telephone and Telegraph Corporation to advise and assist the Greek government in reorganizing, reconstructing, and improving present facilities. All existing public telecommunication systems will be operated by a single autonomous company to be known as The Organization of Telecommunications of Greece. It will be subject to governmental regulation only as to charges for services and methods of financing.

Several systems will be merged in the new company, and they include the Ministry of Posts, Telegraphs, and Telephones, now maintaining long-distance telephone and telegraph networks; Hellenic Telephone Company, now operating about 54,000 automatic telephone lines in urban centers having exchanges of 100 lines or more; and the telephone system of the Island of Rhodes.

The Hellenic Telephone Company was formerly controlled by German interests and the Rhodes system by Italians. Both systems were acquired by the Greek government following the second world war.

Group-Start Method of Subscriber's Line Identification

By F. H. BRAY, D. H. ORMROD, and M. T. WILSON

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IDENTIFICATION of a calling subscriber's line is an essential feature of automatic ticketing and is of primary importance for other applications such as number checking on trunk and toll calls and for the tracing of malicious calls. This article describes the group-start method of line identification applied to step-by-step equipments working into sleeve-control trunk boards. Other methods of line identification have been developed and some are in use, but it is thought that this scheme possesses such advantages as speed of operation, economy in equipment, small modification to existing exchange equipment, adaptability to various automatic systems, simplicity of maintenance, no interference to speech during identification, and freedom from interference by the subscriber.

1. General Plan

Figure 1 is a block schematic of the system as developed for automatic number checking on calls to trunk operators. The method of identification is based on the segregation of the subscribers' lines into groups of 100, each group being provided with a group-start relay and a uniselector. Operation of the group-start relay causes the uniselector to hunt for the line to be identified in the group, and also indicates the thousands and hundreds digits of the group. The

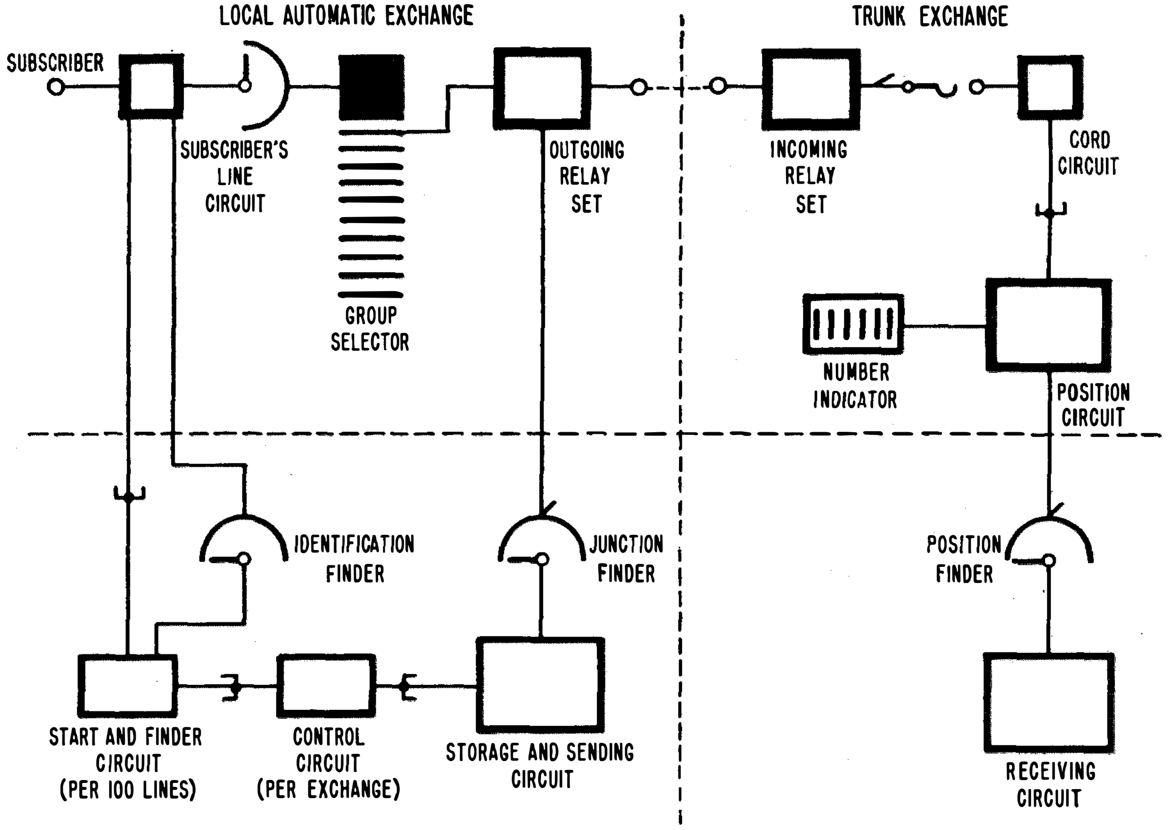


Figure 1—Line-identification trunking diagram for non-register areas.

final position of the uniselector indicates the tens and units digits.

The subscriber at the local exchange dials, say, *O* and is connected via an outgoing relay set to an incoming relay set at the trunk exchange. The operator answers and obtains details of the call. To verify the calling subscriber's number, she throws the identification key associated with the position circuit and a free receiving circuit is associated with the position circuit. An identification start signal is transmitted over the junction causing the outgoing relay set to be connected to a free storage and sending circuit. If the control circuit is free, an identification marking signal is applied by the storage and sending circuit to the private or third wire of the subscriber's line circuit. This marking potential operates the relevant group-start relay, and the uniselector, under the control of the control circuit, hunts for the marked line. Information relative to the subscriber's number is then passed to the storage and sending circuit where it is stored. The start and finder circuit and the control circuit are then released and are available for other calls requiring identification.

The stored information is transmitted in the form of 50-cycle alternating-current impulses, simplexed over the junction to the receiving circuit where they are converted into direct-current im-

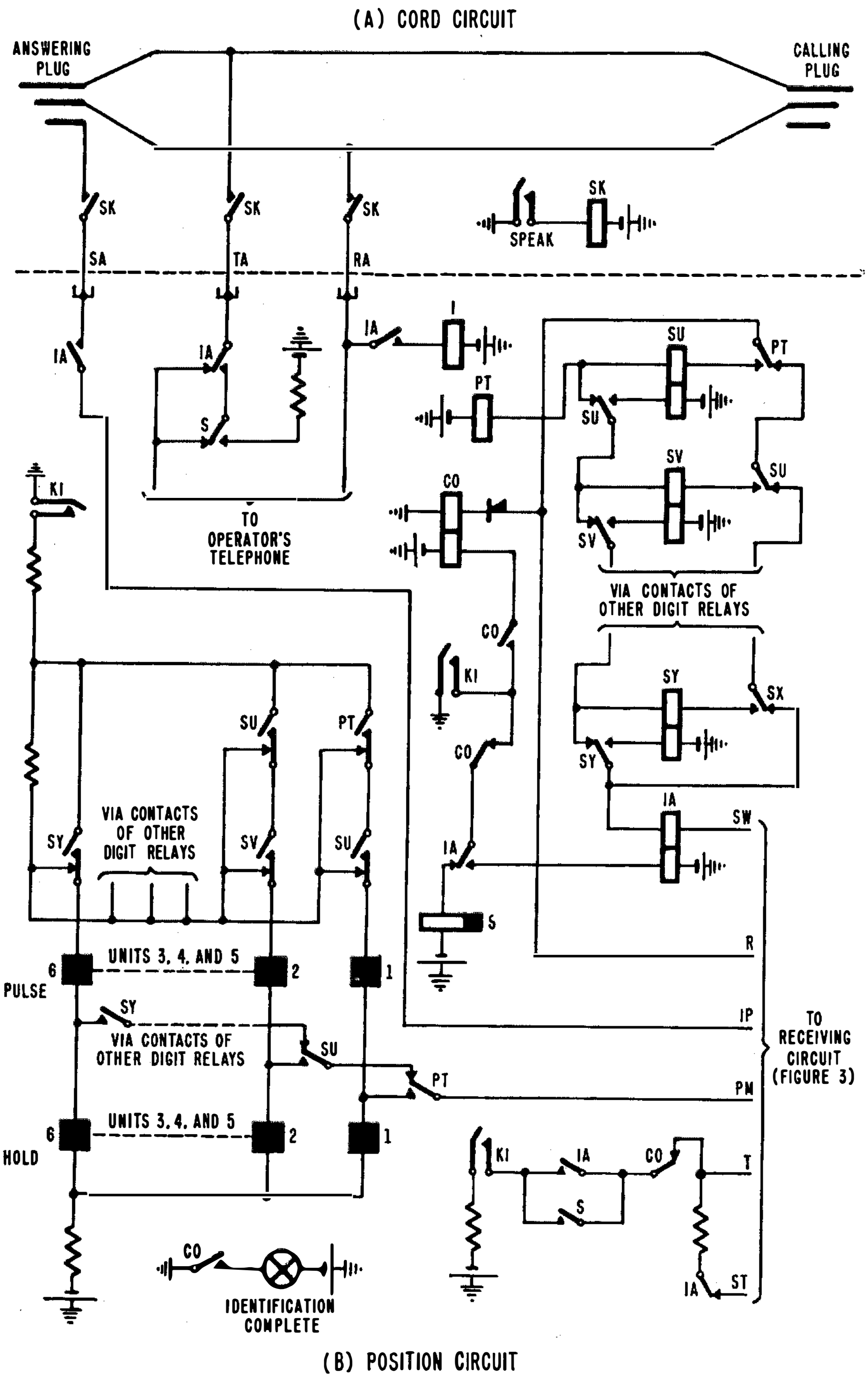


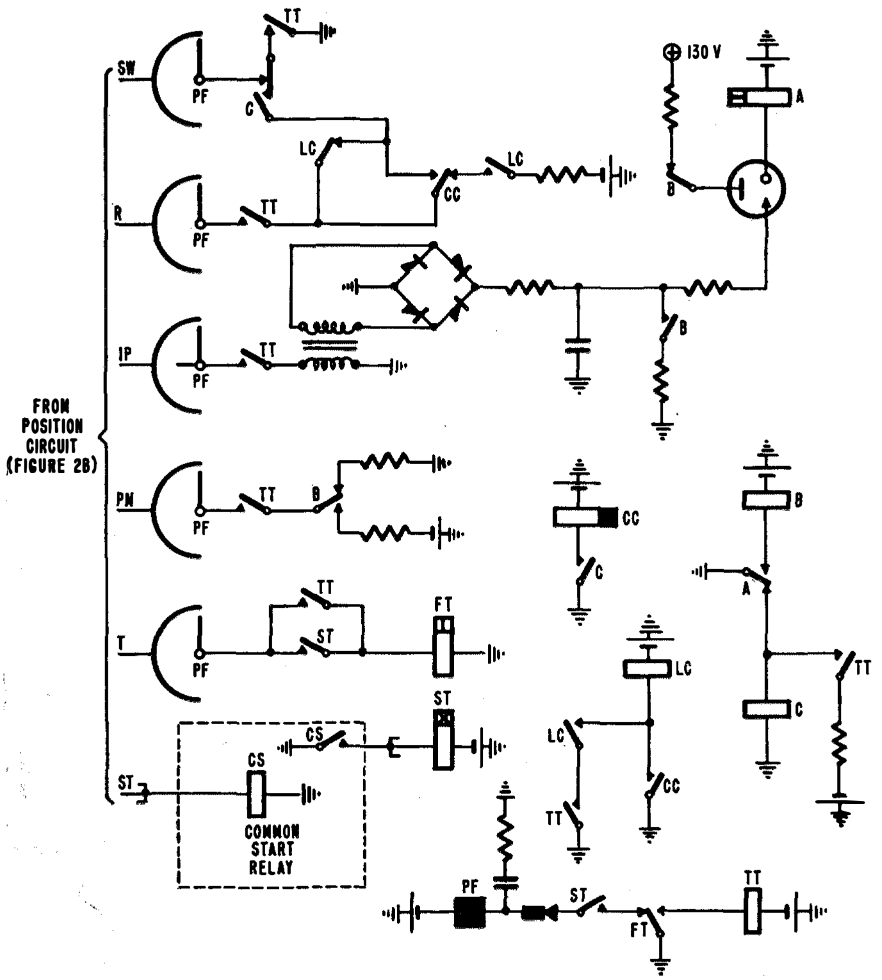
Figure 2—Manual switchboard circuits. Part of the cord circuit is shown above the dotted line at *A*, and part of the position circuit is below it at *B*.

pulses to operate the number indicator, on the operator's position, which displays the calling subscriber's number. The storage and sending circuit and receiving circuit are released after the last digit has been transmitted and displayed, the number indicator being locked under control of the position identification key. The main function of the control circuit is to ensure that only one line in an exchange can be marked for identification at any one time. It is possible to adopt this method, thus eliminating any possibility of cross connections, since the maximum holding time of the control circuit is only 1 second. The modifications required to the standard exchange equipment are very small and are obvious from the more-detailed description that follows.

2. *Initiation of Identification*

2.1 Identification Start Signal

The operator answers an incoming call by plugging a free cord into the answering jack and throwing the cord speak key (Figure 2A), which associates the position circuit, shown in part in 2B, with the connection. To initiate identification of the calling subscriber's number, the identification key *KI* in the position circuit is thrown. Relay *S* operates and applies a start signal to the common start relay *CS* in Figure 3. This operates *ST* in all disengaged receiving circuits and the position finders *PF* hunt to find the calling position. *FT* of the successful circuit operates, breaking the drive of *PF* and operating *TT*. Earth on the *SW* lead operates *IA* and *PT* in the position circuit. *IA* releases *CS* and all other finders stop hunting. *IA* applies a negative potential to the ring lead of the cord circuit to operate *IC* in the incoming relay set (Figure 4) and also releases *S*. During the release time of *S*, a negative potential is applied to the tip lead of the cord circuit to operate *RR*. *RR* applies a pulse of positive polarity to the positive leg of the junction, operating *IS* in the outgoing relay set (Figure 5). Once *IS* is



RECEIVING CIRCUIT

Figure 3—Receiving circuit.

operated and locked, subsequent operations are immune from interference by the calling subscriber, and identification proceeds even should the subscriber replace his receiver.

2.2 Seizure of Storage and Sending Circuit

IS operates the common-start relay *S* (Figure 6) and all free junction finders *JF* hunt for the outgoing relay set in a manner similar to that described in 2.1. Relays *FT* and *TA* operate. *TA* operates relay *D* in the outgoing relay set (Figure 5), which releases *S* and all other junction finders cease hunting.

3. *Group and Line-Marking Signals*

3.1 Method of Applying Marking Signals

It was stated in Section 1 that the lines are segregated in groups of 100. In developing the scheme for use in step-by-step systems using positive-battery metering, advantage was taken of the presence of a rectifier in the meter lead of the subscriber's line circuit. The positive side of the rectifier is disconnected from direct earth and connected in common with the rectifiers of all lines in the group to earth via the low-resistance group-start-relay circuit. The start relay operates to a low positive potential of such a value that there is no possibility of the subscriber's meter operating. This positive potential is also used to mark the *P* wire of the calling line.

In booster-metering systems, it is necessary to add a miniature rectifier per line and to use a tone of about 500 cycles per second as the group-marking signal and a low negative potential on the *P* wire as the line-marking signal. A positive marking signal cannot be used in this case due to the operating limits of the subscriber's meters.

The necessary slight modifications to the identification circuits for these two cases, which

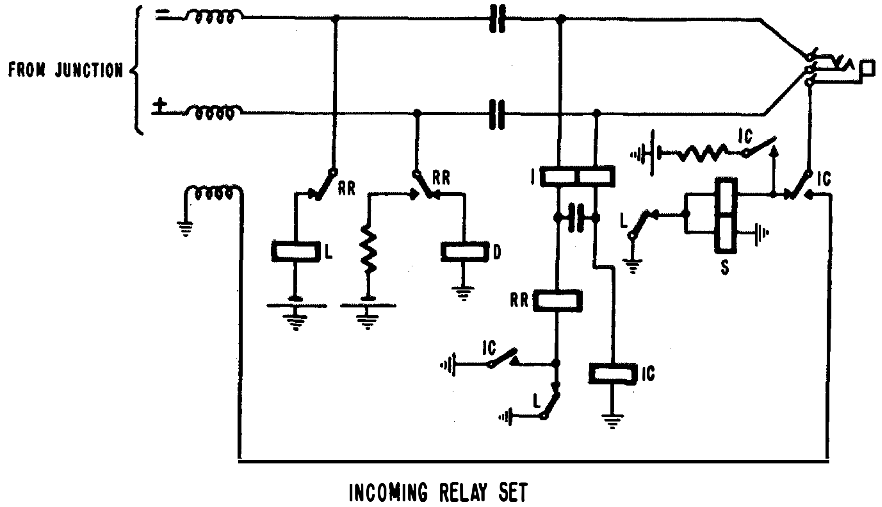


Figure 4—Part of incoming relay set.

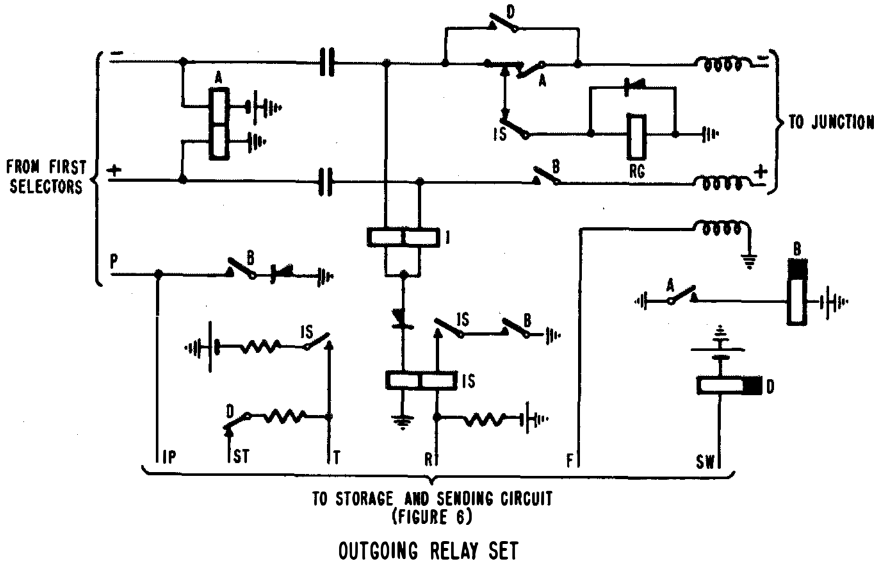


Figure 5—Part of outgoing relay set.

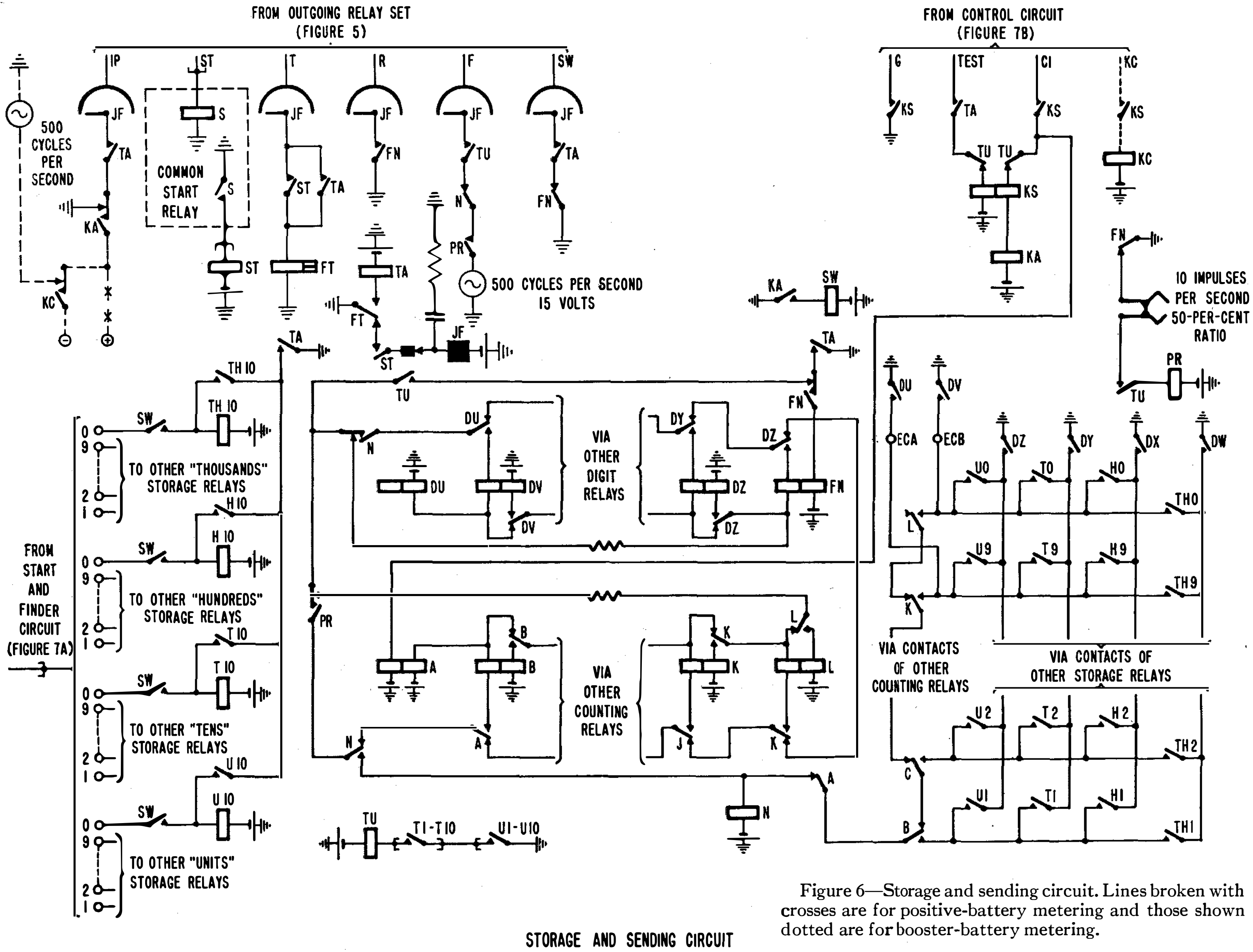


Figure 6—Storage and sending circuit. Lines broken with crosses are for positive-battery metering and those shown dotted are for booster-battery metering.

are those most used by the British Post Office, are indicated in the subsequent descriptions.

It is apparent that the positive-battery-metering scheme can be easily adapted for use on 4th-wire metering or flat-rate systems and is equally applicable to line-finder or line-switch equipments.

3.2 Seizure of Control Circuit

Figure 7B shows the alternative connections for the cases mentioned under Section 3.1; connections shown with crosses breaking the lines are used for positive-battery-metering systems and those shown dotted are for booster-battery metering. Similar conventions are used in Figure 6. Only one control circuit is provided per exchange for the reason given in Section 1. A standby circuit is always provided to cover any fault liability and to assist maintenance. Assuming the control circuit is not in use when relay *TA* of Figure 6 operates, *KS* is operated via the test lead and *G* operates in the control circuit. *G* operates *KA* and holds *KS* on the *CI* lead. *KA* operates *SW*.

In a positive-battery-metering exchange, the operation of *KA* applies the start and marking positive potential of about 14 volts to the *IP* lead of the outgoing relay set. The *IP* lead is connected to the *P* wire of the established connection

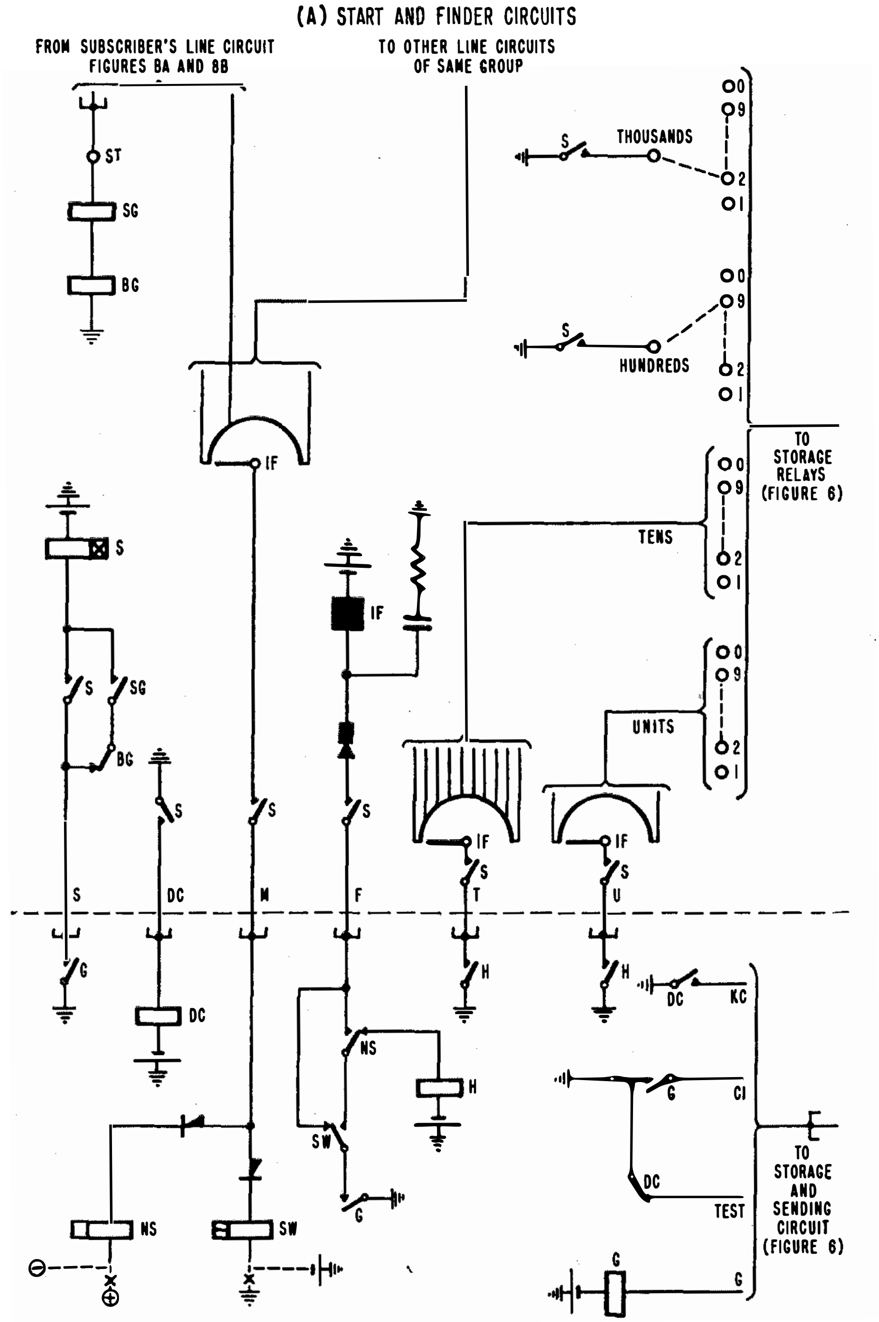


Figure 7—Line-identification start and finder circuit (per 100 lines) at *A* above the dotted line, and control circuit (per exchange) at *B* below line. Lines broken with crosses are for positive-battery metering and those shown dotted are for booster-battery metering.

and this positive potential maintains the busy and holding condition of the connection. In the case of the booster-metering line circuit, *KA* applies the group-marking signal of 500 cycles per second to the *P* wire. The line-marking signal for this case is about 6 volts negative and is described later.

The operation of *G* in the control circuit prevents any other storage and sending circuit from originating the marking signals since the operating earth for *KS* is removed from the test lead.

4. *Identification of Calling Subscriber's Line*

4.1 Seizure of Start and Finder Circuit (Positive-Battery Metering)

The low positive potential on the *P* wire from the storage and sending circuit (Figure 6) appears on the subscriber's meter lead (Figure 8) and operates *SG* relay in the group-start circuit (Figure 7) in series with the subscriber's meter and relay *BG*, but the meter and *BG* do not operate. The *BG* relay is the "meter-guard" relay and operates, together with *SG*, only when any line in a group is being metered. Identification is delayed whilst *BG* is operated.

The operation of *SG* to the group-marking signal operates and locks *S* in the identification finder circuit (Figure 7) so that once a marking signal has been received it is not affected by any

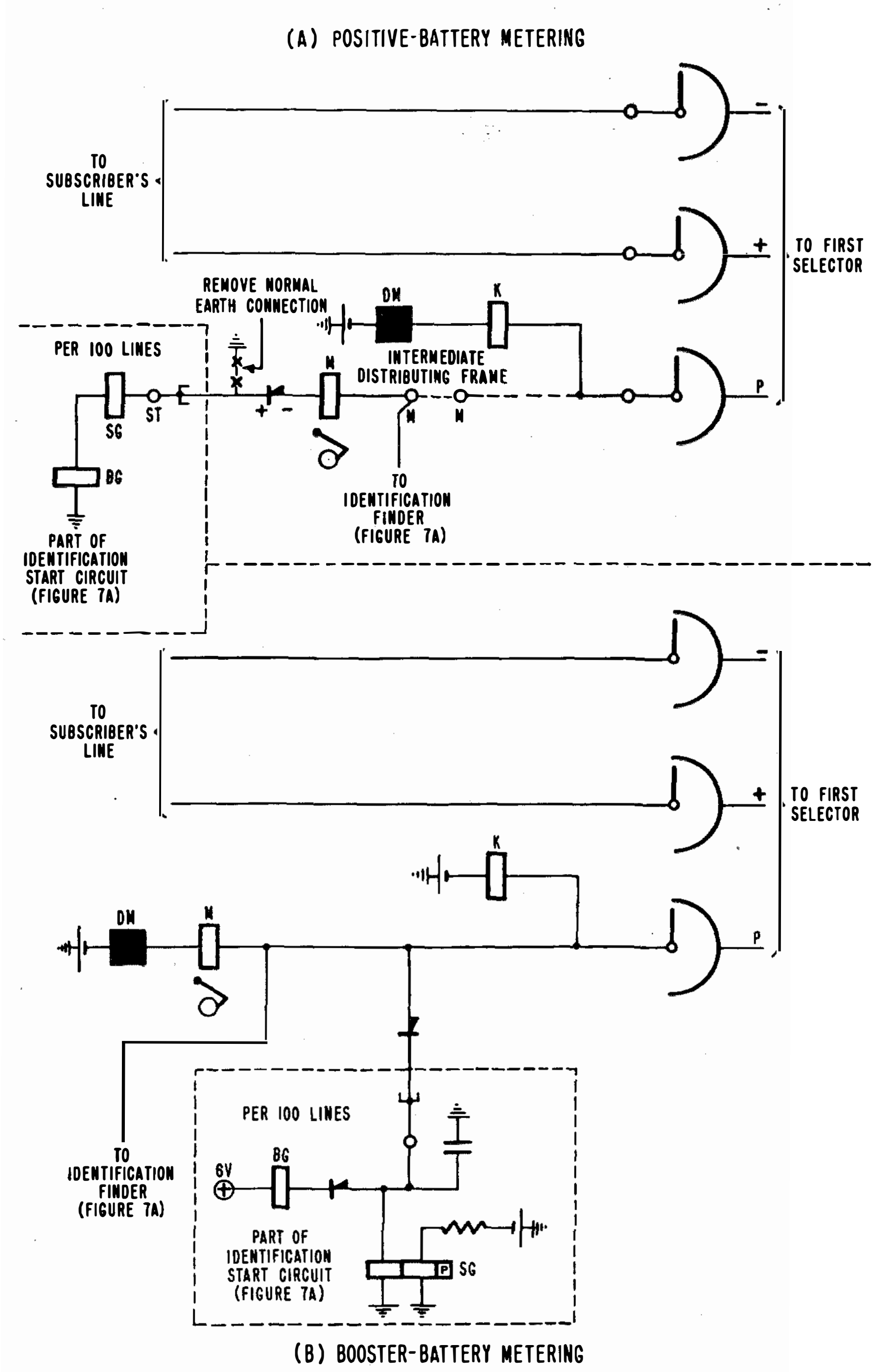


Figure 8—Connection of subscriber's line circuit to identification equipment.

subsequent metering signals to other subscribers' lines in the group. If the marking signal is received while another line in the group is being metered, the operation of *S* is delayed for the metering period, approximately 200 milli-seconds. When *BG* releases, *SG* remains operated to initiate identification.

4.2 Seizure of Start and Finder Circuit (Booster-Battery Metering)

The 500-cycle group-marking signal is applied in Figure 8B via a miniature rectifier to the common start lead. The rectified current operates a polarized relay *SG* which functions as described under 4.1. The connection of *BG* relay to 6 volts positive and its connection to the *ST* lead via a rectifier ensures that it operates only when a metering signal is present on any line in the group. *BG* performs the same functions as described under 4.1.

4.3 Method of Identifying Calling Subscriber's Number

When the start relay *S* (Figure 7) is operated, the identification finder *IF* searches for the calling line whose *P* wire is connected to the marking potential. Relay *DC* is also operated. The identification leads are connected direct to the *P* wires of the subscriber's line circuits in the group and arranged in pairs round the banks of a 50-point uniselector so that 100 lines can be accommodated by one switch. There are two identical test circuits in the control circuit, only one of which is shown on the *M* lead as relays *SW* and *NS*. Two lines are tested at once so that the maximum number of steps that the uniselector has to hunt over is 49, and the maximum searching time is, therefore, less than one second.

When *IF* reaches the marked line, *SW* operates, disconnects the circuit of *IF* and operates the switching relay *H*. Since *SW* will also operate when a line in the metering condition is encountered, relay *NS* is provided and connected to the same potential as the marking signal. *NS* operates to the metering signal and maintains the drive circuit for *IF*, so preventing the selector stopping on any line other than the one marked by the line-marking signal. Provision is made to prevent the second test circuit driving *IF* past the marked line if the metering condition exists on the opposite line.

The final location of *IF* indicates the tens and units digits of the calling subscriber's number and by suitable strapping on the banks the appropriate storage relays in the storage and sending circuit (Figure 6) are operated. Since other digits comprising the subscriber's number are common for all lines in the group, they can obviously be indicated by contacts of the *S* relay connected direct to the appropriate storage-relay terminals. If the calling line is one of a private-branch-exchange group, then it is a simple matter by strapping the contact on the *IF* bank to operate the storage relays to give either the directory number of this line, or its actual exchange number.

In the booster-metering case (Figure 8B), the method of identification is similar, the only difference being that the marking potential is a low negative potential (about 6 volts) instead of positive. When *DC* operates, it operates *KC* in the storage and sending circuit (Figure 6). *KC* is fitted only in booster-metering exchanges. The operation of *KC* replaces the 500-cycle group-marking signal with the low negative line-marking signal, which operates *SW* as already described. *SW* and *NS* are connected in the alternative manner as shown in Figure 7.

4.4 Release of Control and Start and Finder Circuits

When the complete number is stored, as indicated, by the operation of the tens and units storage relays, *TU* (Figure 6) operates and releases *KS* and *KA*. *KS* releases *G* in the control circuit and *KA* removes the marking potential from the *P* wire via *IP*. The control circuit and the start and finder circuit are released and are immediately available for other circuits requiring identification.

5. *Transmission of the Calling Subscriber's Number*

5.1 Method of Transmitting and Counting Digits

The sending portion of the storage and sending circuit (Figure 6) consists essentially of an impulse generator, an impulse-counting train, and a digit-distribution train. The impulses are generated by a capacitor-controlled relay and repeated by relay *PR* as impulses of 50 cycles per second at a speed of 10 impulses per second and with a make-to-break ratio of approximately 1:1. The train of impulse-counting relays *A* to *L* is controlled by *PR* and the digit-distribution train *DU* to *DZ* is controlled by an end-of-digit relay *N*. Contacts of the storage relays and the digit-distribution relays are arranged in a co-ordinate marking field in the well-known manner.

5.2 Exchange Code Digits

In a multi-exchange area, each exchange will have a characteristic code consisting of, say, 2 digits and this code can be indicated by common equipment. The storage and sending circuit is arranged to provide this indication by direct cross connection of the contacts of the first 2 relays of the digit-distribution train, *DU* and *DV*, to the marking field. This characteristic code is shown in Figure 6 as 90.

5.3 Counting the Impulses and Digits

Relay *A* is pre-operated in parallel with *KA*, and when *TU* operates, *A* is held on its other winding. *DU* operates and earths the marking field to mark the first digit of the exchange characteristic code. On the first operation of *PR* relay *B* is operated in series with *A* and when *PR* releases, *A* releases but relay *B* is held. *B* operated with *A* released, indicates that one impulse has been transmitted. On the second operation of *PR*, relay *C* in the counting train operates in series with *B* and when *PR* releases, *C* is held but *B* is released. This operation continues throughout the train until the first digit, in this case *9*, has been sent when *K* operates. On the subsequent release of *PR*, relay *J* releases and *N* operates to earth from *DU*. *N* opens the pulsing-out circuit and *DV* operates in series with *DU*. A subsidiary counting-train circuit, which is not shown, counts the next two cycles of *PR* to give an inter-digital pause of approximately 200 milli-seconds. This results in the release of *N* followed by *DU*. *DV* is held and marks the second digit to be transmitted. This digit is sent and the impulses counted in exactly the same manner followed by the numerical digits comprising the subscriber's number. The distribution relays *DW*, *DX*, *DY*, and *DZ* operate in sequence to mark the thousands, hundreds, tens, and units digits respectively. The number of impulses sent for each digit is determined by the relevant storage relays that are operated. *FN* operates when the last digit has been transmitted.

5.4 Release of the Storage and Sending Circuit

The earth applied by *FN* to the *R* lead of the outgoing relay set (Figure 5), short circuits relay *IS*, which releases and removes the test potential from the *T* lead. *FT* and *TA* relays in the storage and sending circuit (Figure 6) release followed by the release of all other relays that may be operated. This circuit is now free to be seized by any other outgoing relay set that may be waiting to originate identification.

5.5 Simplexing

To avoid interference to speech whilst impulses are being transmitted to the trunk exchange, simplexing coils are inserted at both ends of the junction. Referring to Figure 5, the alternating-current impulses applied to the third winding on the *F* lead are induced into the line windings in parallel. Since no voltage is developed across the line, the alternating-current impulses cause no interference to either the subscriber or operator during conversation. At the incoming end (Figure 4), impulses are induced from the line windings into the third winding and passed to the sleeve connection of the jack of the incoming relay set.

6. *Reception and Display of the Number*

6.1 Impulse Receiving Circuit

The received alternating-current impulses pass via the sleeve of the answering plug (Figure 2A), the *SA* lead of the position circuit, the *IP* lead of the receiving circuit (Figure 3) to the primary winding of a step-up transformer, the secondary being connected to a bridge rectifier. The output from the simplexing coils is kept at a low impedance to reduce the effect of stray line disturbances.

The received impulses, now converted to direct-current pulses, are applied to the control electrode of a cold-cathode tube via a resistor-capacitor combination that imposes a time delay of approximately 20 milli-seconds. This provides a guard facility against stray line disturbances triggering the tube and being registered as impulses. When the tube strikes on receipt of an impulse, the cathode relay *A* operates followed by *B*. Due to the removal of the short circuit, *C* operates followed by *CC* and *LC*. The *C* relay holds during impulsing but releases during the inter-digital pause whereas *CC* holds during that period. The operation of *B* extinguishes the tube and *A* releases. *B* follows the impulsing of *A* and repeats the received impulses as alternate battery and earth pulses to the *PM* lead. The circuit is so arranged that *B* delivers pulses the make period of which is kept approximately constant independent of the speed of received impulses.

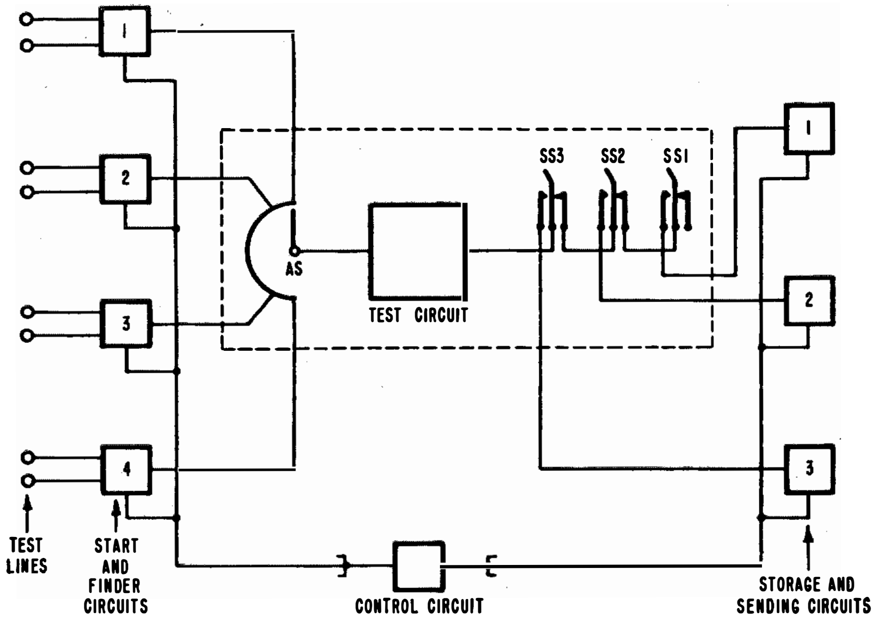


Figure 9—Routine-testing arrangements.

6.2 Number Display

The calling subscriber's number is displayed on a number indicator mounted on the operator's position key shelf. The indicator consists of a number of identical units mounted under a common cover. Each unit consists of a rotor bearing numbers 1 to 0 on its periphery and capable of being set and held in any position by two electromagnets. These magnets respond in turn to the alternate battery and earth pulses on the *PM* lead, and on the termination of a train of impulses corresponding to the number, both magnets are left energised in series under control of the position identification key. A digit-distribution train of relays *PT* to *SY* (Figure 2B) is provided to route digital impulses to each unit in turn, these relays being operated and released in sequence by the release of *C* (Figure 3) during the inter-digital pause.

6.3 Release of Impulse Receiving Circuit

When all digits have been received, relay *CC* releases followed by *LC*. During the release time of *LC*, a positive battery pulse on the *R* lead operates *CO* which locks. *CO* removes the hold potential on the *T* lead and the receiving circuit is released.

6.4 Release of Number Display

At this stage, all the identification equipment both at the manual exchange and the distant automatic exchange has released, the conversational equipment is restored to the condition prior to identification and the number is displayed to the operator. Restoration of the position identification key allows the number indicator to restore to normal.

An identification check may be repeated at any time so long as the calling subscriber remains on the line.

7. *Routine Testing*

Figure 9 shows the arrangements for testing the identification equipment. By means of selecting keys *SS1*, *SS2*, etc., any storage and sending circuit can be connected to the routine test circuit and manually tested. Any storage and sending circuit can also be connected automatically to each start and finder circuit in turn via access switches *AS*. Two test lines, which are not necessarily spare lines, are selected in each start and finder circuit, one in the first 50 lines of the group, and the other in the second 50 lines. By a suitable selection of test lines, it is possible to test all the storage and counting relays in each storage and sending circuit. Provision is also made to routine continuously any start and finder circuit.

In addition to the main test circuit, a manually operated tester is provided for testing the control circuit. This tester is permanently wired to a spare jack position into which the standby control circuit is normally fitted.

8. *Equipment*

8.1 Start and Finder Circuits

The start and finder circuits are assembled in units of 200 lines as shown in Figure 10. There are 2 circuits per unit and 10 such units, sufficient for 2000 lines, mount on a rack 10 feet 6 inches by 2 feet 9 inches. This arrangement of unit mounting becomes very useful for cases of future extensions to exchanges.

8.2 Common Equipment

The rest of the identification equipment comprising storage and sending circuits, control circuit, test circuit, and miscellaneous alarm circuits, etc., is mounted on a rack 10 feet 6 inches by 4 feet 6 inches and is sufficient for a 10,000-line exchange.

8.3 Number Indicator

The number indicator, as mentioned in Section 6.2, is mounted on the key shelf of the operator's position. It consists of a number of identical units one for each digit to be displayed, mounted under a common cover and plugging into position. Figures 11 and 12 show two views of the number indicator. The indicator mounts on the key shelf with only the upper portion protruding above the level of the shelf.

Briefly, the operation of a unit is as follows. Each unit is essentially a type of 2-phase motor consisting of a rotor with projection teeth bearing the numerals 1–0 on its periphery. The teeth are so disposed that they can be attracted by the fields of 2 pairs of magnets known as the pulse and hold magnets respectively, fixed to the frame of the unit inside the rotor. When the indicator is first connected to the impulsing

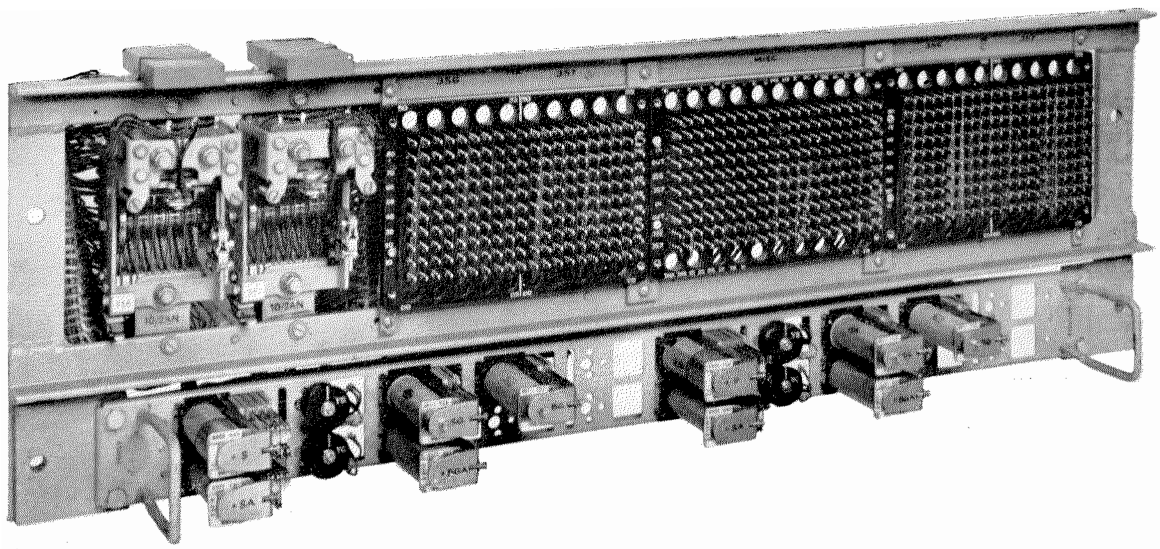


Figure 10—An assembly of start and finder circuits for 200 lines.

circuit, the hold magnet is energised, but has no effect other than to move the rotor slightly off normal. On the first operation of the impulse contacts, the pulse magnet is energised and the

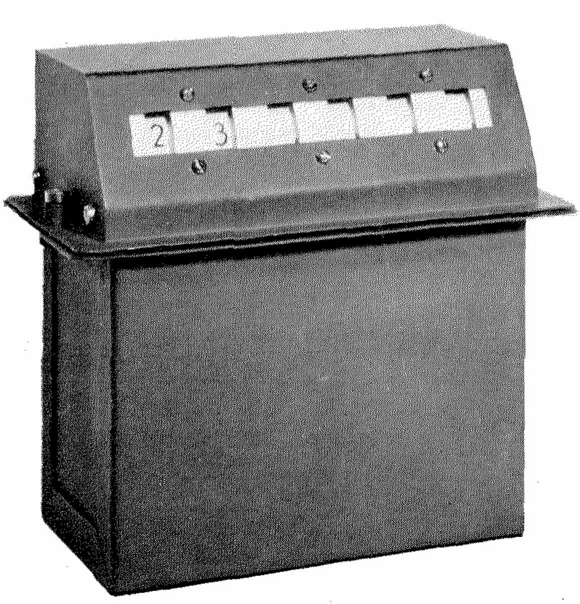


Figure 11—Number indicator. Only the upper section protrudes above the operator's key shelf.

hold magnet short circuited. The rotor moves the equivalent of a half step and when the impulse contacts restore, the hold magnet is energised, the pulse magnet is short circuited and the rotor completes a full step. This sequence of operations is repeated until the complete digit has been registered and the unit is left with both pairs of magnets energised in series. A small latching device operable by each magnet prevents the release of the rotor during impulsing and holds the rotor in position when impulsing has finished. The nominal operating speed of the indicator is 10 impulses per second and it operates reliably to minimum pulses of 30 milliseconds. It has been widely used to date for miscellaneous remote-control applications and in aircraft for giving various indications to the pilot.

9. *Application to Register-Controlled Systems*

This article has dealt with the group-start method of line identification applied to straightforward step-by-step systems. It can be easily applied to register systems as is shown by the following brief description of its application to the director system used by the British Post Office. In this application, advantage is taken of the fact that the numerical storage circuits of the director normally used for storing the numerical digits of the wanted subscriber are not used on *TRU*, *TOL*, etc., calls and can, therefore, be used to store the numerical digits of the calling subscriber's number.

Figure 13 shows a block trunking diagram of the suggested scheme. When the calling subscriber dials, say, *TRU* for a trunk call, the *BC* selector in the director is positioned and a relay operates and causes the number-storage circuit to become associated with the director and sends the identification-marking signals to the calling

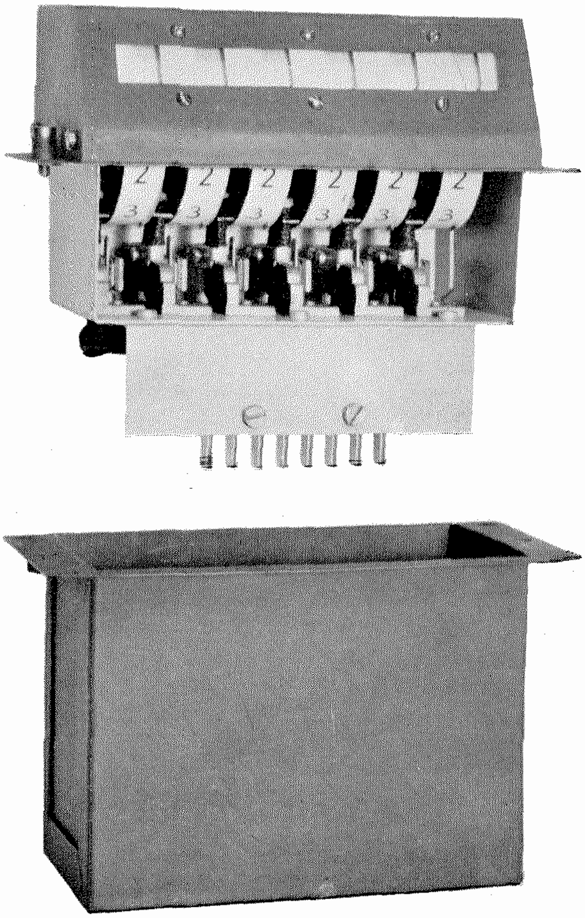


Figure 12—Number indicator with lower case removed and showing the plug arrangement of the connections.

subscriber's line in a manner similar to that already described. The identification finder in the

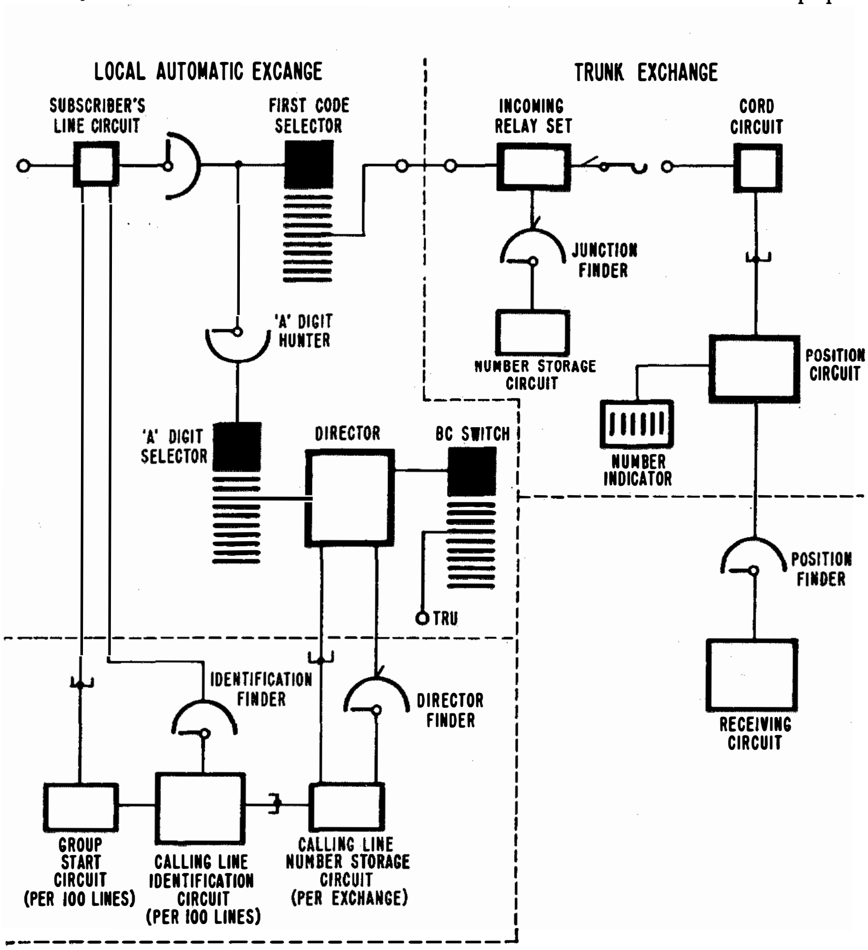


Figure 13—Trunking diagram for line identification as used in register exchanges.

start circuit locates the calling line and the number of the line is marked in the number-storage circuit, which then transmits to all 4 number-storage circuits in the director simultaneously. The start and finder circuit and the number-storage circuit then release, the holding time of this equipment being approximately 1 second. The director transmits the necessary routing digits to reach the trunk exchange, while identification is takingplace. A number-storage circuit is then associated with the trunk incoming relay set, and a signal is sent back to the director, which now transmits digits indicative of the calling exchange if required, followed by the calling subscriber's number. The director releases in the usual manner and the calling signal is given to the trunk operator when storage is complete. When the operator answers, she throws the position identification key, all the information is extracted relating to the calling number and displayed on a number indicator in the manner already described.

10. Conclusion

Tests so far made indicate that the system operates reliably, the circuits involved being simple and robust, and that adequate maintenance-test facilities have been provided.

Pulse Modulation*

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THE BROAD SIGNIFICANCE of pulse modulation is dealt with from its original concept through the various methods of attaining it. Its application to time-division multichannel systems is considered. More recent developments in pulse-count-modulation systems, and also potential applications to switching problems, are described.

• • •

1. *Origin of Pulse Modulation*

The classical concept of transmission of speech by electrical means involves converting the sound vibrations into currents, one characteristic of which, amplitude or phase, is made to vary according to the sound intensity as a continuous curve versus time and an inverse operation at the receiving end. In the last few years, however, other methods have been used, in which the transmission of any speech channel occupies but a fraction of the time, the information being transmitted by samples or current pulses recur-

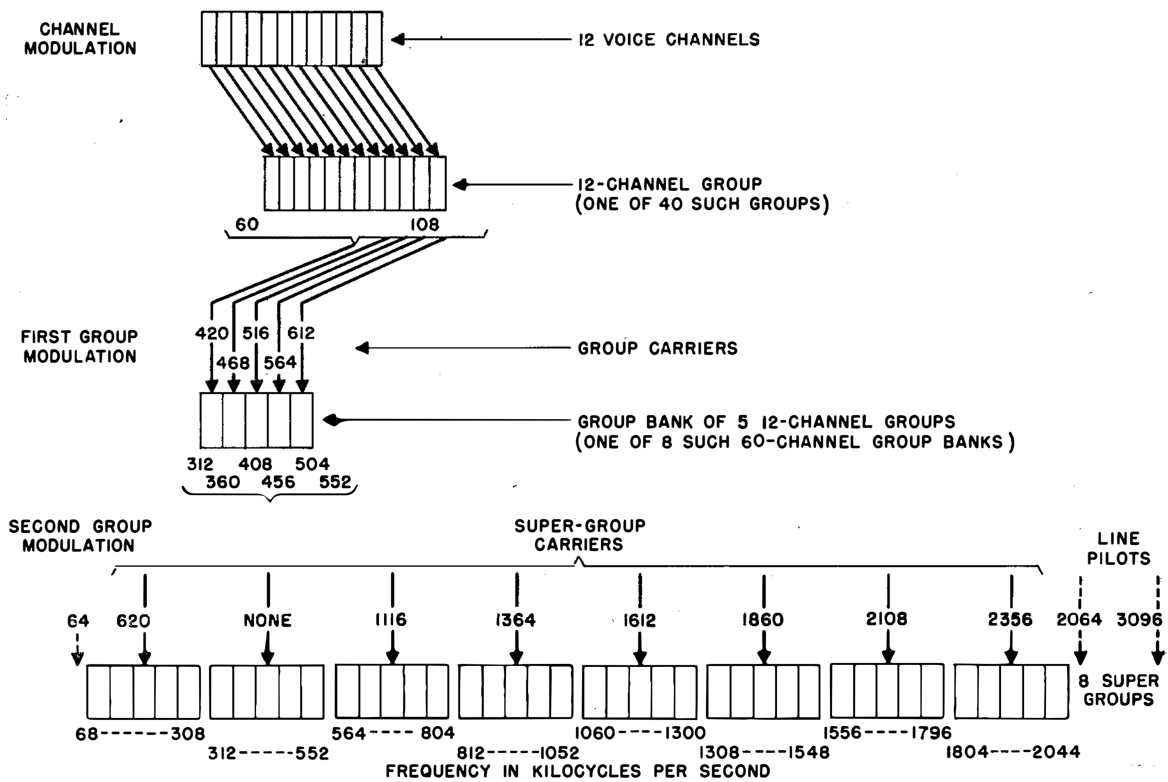

Figure 1—Typical layout of a multiplex transmission system by frequency division.

* Reprinted from *Proceedings of the I.R.E.*, v. 37, pp. 702–705; June, 1949. Presented at the Institute of Radio Engineers National Convention, New York, New York, March 24, 1948.

ring at a suitable rate. These methods are known under the general name of "pulse modulation."

The early publications on pulse modulation are found mostly in the form of patents.[1] A review of these indicates that up to 1935 the thinking of the authors was very often in terms of improving the power efficiency of the transmission system, either at the transmitter or in the link.

Since 1935, however, more and more consideration has been given to pulse modulation as a method of multiplexing a number of telephone channels.

The accepted technique in multichannelling has been to take each individual telephone frequency band and translate it to successive positions in the spectrum of frequencies by means of modulators and filters, transmit this information on a single path, and perform the inverse operation at the other end of the link. The bandwidth utilized on the link increases, of course, in direct proportion to the number of channels. A typical layout of a multichannel transmission system by frequency division is shown in Figure 1.

The technical literature on these subjects at this time is very abundant. These elements have reached a high degree of perfection and practicability.

Terminal equipments were thus realized that were capable of reproducing the speech in any individual channel with high fidelity. The other parts of the transmission system had to be improved accordingly, and rather severe requirements had to be placed on them as to limitation of nonlinear distortion to avoid cross talk between channels, stability of characteristics, and noise level.

[1] Among such patents can be cited:

Name	United States Patent Number	Filing Date
R. A. Heising	1,655,543	April 18, 1924
R. D. Kell	2,061,734	September 29, 1934
K. Posthumus and C. G. A. VonLindern	2,161,087	December 17, 1935
A. M. Nilcolson	2,021,743	June 13, 1930
R. M. Ranger	1,873,786	September 29, 1928
A. S. Riggs	2,048,081	April 29, 1933
R. E. Shelby	2,171,150	September 14, 1936
D. G. C. Luck	2,113,214	October 29, 1936
G. Bozzi (Italian Patent)	348,656	February 8, 1937

While fully acknowledging the remarkable success of the frequency-division transmission system, Reeves and others[2] took an interest in 1936 and the following years in the possible merits of multichannel systems based on the con-

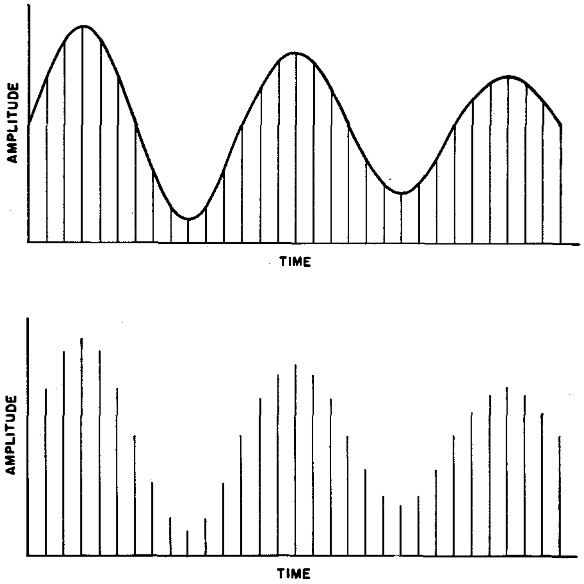

Figure 2—Sampling of speech.

cept of time distribution, translating into the telephone art some of the well-established practices of multichannel telegraphy. The attractive feature of the idea was that such methods might prove especially useful where it is difficult or expensive to attain the quality of characteristics required for reliable multichannel transmission on a frequency-division basis.

2. *Application of Pulse Modulation to Time Multiplexing*

The basis of pulse-modulation time multiplexing is the fact that it is not necessary to transmit the complex wave form of a speech wave in its entirety. It is sufficient to take successive "samples" of the amplitude in the channel at separate time intervals and transmit this information as a series of pulses carrying the information, as indicated in Figure 2. If the number of samples per second exceeds twice the highest frequency to be transmitted in the voice band, the

[2] E. M. Deloraine and A. H. Reeves, United States Patent 2,262,838, filed, November 8, 1938.

telephone channel can be correctly reproduced at the distant end.

The bandwidth required in the transmission medium through which the pulses are propagated depends on the shape of these sampling pulses, and the method adopted for carrying the information on pulses.

A number of incoming telephone channels can be sampled at time intervals that are displaced in the time scale just enough to avoid the superposition in time of any two pulses. The successive samples of telephone channel 1 will be followed by the successive samples of telephone channel 2 and so on until the whole time interval between two samples of channel 1 is completely filled, as shown in Figure 3.

It is clear, in consequence, that the number of channels that can be transmitted over a single link increases as the individual samples are made shorter. If the time is used in the most efficient manner, the bandwidth required for the transmission of n channels is close to n times the bandwidth of an individual channel, as is the case for multichannel systems utilizing frequency division.

3. *Methods of Pulse Modulation*

3.1 Pulse-Amplitude Modulation

The samples of speech amplitude can be transmitted as such. This is pulse-amplitude modulation. This method is efficient in utilization of bandwidth. The requirements on the link for nonlinear distortion are, however, less stringent than for the frequency-division method, since, at a given time, one channel only is present and cross talk cannot be introduced in this manner, though it can reappear in a different manner if the pulses are lengthened by the transmission link so as to overlap each other to a certain extent. The requirements as to link stability and low noise are not changed materially.

It is the consideration of signal-to-noise-ratio improvement, particularly for radio links, that drew attention to other possible methods of pulse modulation.

3.2 Pulse-Width Modulation

One of the first methods to be considered was pulse-width modulation, which inherently requires more bandwidth than pulse amplitude modulation, but provides a means to trade that increase in bandwidth for an improvement in the signal-to-noise ratio. Pulse-width modulation is not, however, economical or efficient in power insofar as the significant information is, in fact, the timing of the beginning and the end of individual pulses, and this led to the next method.

3.3 Pulse-Time Modulation

It was suggested that pulses of constant amplitude, form, and duration should be used, these pulses being displaced in time from a uniform spacing to an extent corresponding to the amplitude of the sample. This is more efficient than transmitting the beginning and end of pulses modulated in width, since the amplitude is translated in time displacement of one pulse, from a reference position which, however, is not transmitted except as a marker for all channels. This is usually referred to as pulse-position or

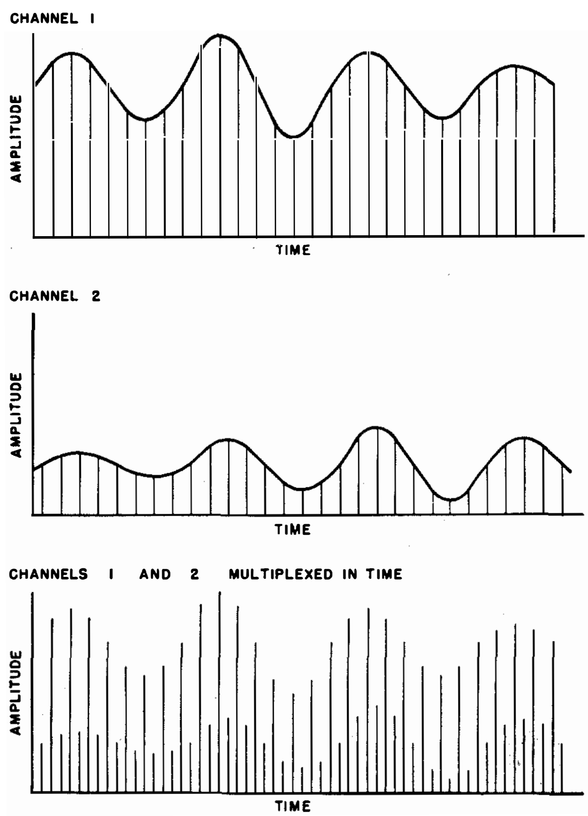

Figure 3—Pulse-time multiplexing.

pulse-time modulation.[3] As compared to pulse-amplitude modulation, the number of channels that can be transmitted over a given bandwidth is reduced. If we assume, for instance, that the

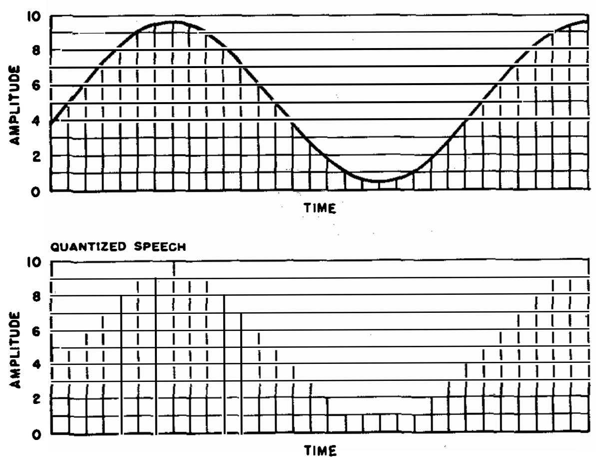

Figure 4—Quantization of sampled speech amplitudes.

individual pulse is time-modulated only to the extent of its own duration, the number of channels is divided by two. But a major advantage is that the link need not have linear amplitude characteristics. The repeaters operate as triggers, with the requirement, however, that they must trigger accurately in time. The possible variations of level in the link are compensated by the trigger action, since the output level of the repeaters is independent of the input level between wide limits. This method provides a means to trade bandwidth for a signal-to-noise ratio comparable with that obtained with frequency modulation. Noise is introduced in the system to the extent only that it will change the timing of the pulse, so that the influence of a given noise amplitude in the link decreases as the pulses become sharper and also as the time displacement due to modulation is increased. This method of improving the signal-to-noise ratio has been found desirable in applications to radio links, though it is paid for by a reduction in the number of channels for a given transmission bandwidth.

[3] A. H. Reeves, French Patent 833,929, filed, June 18, 1937; and addition 49,159, filed, July 5, 1937; and United States Patent 2,266,401, filed, June 9, 1938.

4. *Pulsed Frequency Modulation*

Another pulse-modulation method for radio links in which the pulses remain constant in time position, as well as in amplitude and duration can be designated as pulsed frequency modulation.

The information is transmitted by means of a change in the carrier frequency of each individual pulse, which is made to vary proportionally to the amplitude sample at the corresponding time. The frequency modulation brings in a signal-to-noise improvement, which involves additional requirements in bandwidth. The utilization of the total bandwidth improves, however, as the number of channels is increased, because the additional band required for frequency modulation and a given signal-to-noise improvement becomes a smaller fraction of the bandwidth required for the correct transmission of the pulse shape.

5. *Pulse-Count (or Code) Modulation*

The assimilation of telephone transmission to telegraph transmission becomes still closer in a method suggested by Reeves.[4] Not only is the speech curve sampled, but the amplitude samples are submitted to two successive transformations: quantization, and counting or coding.

By quantization is meant the process by which the amplitude sample is not sent with its exact value but is approximated to one of a finite number of discrete levels, as shown in Figure 4. It is clear that, when such a series of quantized pulses is sent into a low-pass filter cutting off at the maximum speech frequency which it is thought proper to reproduce, the result will show some distortion due to the effect of the difference between the quantized and exact amplitude samples. This distortion obviously will decrease when the number of quanta is increased for any given maximum amplitude.

At the receiving end, a method that discriminates against amplitudes that do not correspond to a quanta is used. This is capable of eliminating forms of speech impairment introduced in the transmission systems, if they can be distinguished from the quanta.

[4] A. H. Reeves, French Patent 852,183, filed, October 3, 1938; and United States Patent 2,272,070, filed. November 22, 1939.

While the advantages of speech quantization do not necessarily apply only to pulse modulation, it is of interest to combine the two processes. However, a third step can be taken with advantage.

Instead of transmitting the series of quantized amplitudes directly over the line or through space, a third transformation is performed, which is counting or coding. The information that it is really necessary to communicate is the number of quanta present in any particular amplitude sample; this can be achieved by the transmission of a group of pulses, n for instance, each one having s possible characteristics. The total number of distinguishable levels will be $q=s^n$, and such a mechanism will thus allow the counting of q different levels. Thus, a group of five pulses of the on-or-off type can be used to count $2^5=32$ quantized levels in a manner similar to what is done in teleprinter signal transmission. In this instance, where $s=2$, the system is a binary one. Other values for s can obviously be utilized, and ternary systems, for instance, have been considered particularly for wire transmission.

The fundamental advantage of binary or ternary systems is that the demodulation process consists in merely acknowledging the presence or absence of a pulse of a definite type at a definite time position. In the case of the binary code, the number of errors will fall to an insignificant value provided the pulse amplitude remains sufficiently larger than the noise root-mean-square amplitude. For instance, for a binary code and pure random noise, a ratio of 18 decibels in root-mean-square signal-to-noise ratio will be sufficient to provide a signal-to-noise ratio in the order of 70 decibels after demodulation.

Because of the fact that noise in the demodulated signal originates in the random distribution of noise bursts above its root-mean-square value, the signal-to-noise improvement follows a very different law from that which it does in pulse-time modulation or frequency modulation.

In all cases, the relation between demodulated and input signal-to-noise ratios can be determined theoretically with a sufficient simplicity in the case when the signal amplitude is noticeably larger than the noise root-mean-square value. Above that threshold, the two ratios vary in direct proportion for pulse-time modulation and frequency modulation. For pulse-count modulation, however, the proportionality holds between input signal-to-noise in power ratio and demodulated signal-to-noise ratio in decibels.[5] Thus, lowering the input power ratio by 3 decibels brings the output ratio down to 35 decibels, and increasing it by 3 decibels brings the output ratio up to 140 decibels.

The effect of noise in the latter case thus becomes entirely negligible. This advantage is paid for by the distortion that quantization brings in and the additional bandwidth that the counting process necessitates. If f is the maximum frequency that it is proposed to transmit by voice channel, and n is the number of pulses per group, then the bandwidth per channel should be somewhat larger than n times f.

6. Time Versus Frequency Translation Multiplexing

The technique of modern telecommunication equipments shows a definite trend toward systems that permit of multiplexing a great number of telephone channels. These channels in turn can be used for telegraph or facsimile communication. Time multiplexing by means of pulse modulation is a solution that presents some interesting features.

One common advantage belonging to all pulse methods is the comparative ease with which any particular channel can be dropped and reinserted at any repeating point. This necessitates a complicated and formidable array of modulating and filtering apparatus when frequency division is utilized. However, in pulse transmission this may be accomplished simply by gating out every nth pulse or group of pulses following the marker signal. Each channel keeps its individuality throughout the link, whereas in frequency-division technique, the economic considerations lead to a definite grouping of the channels.

Pulse transmission eases the requirement for linearity in repeater equipments. This advantage is less marked on cables where very linear amplifiers are available and where propagation char-

[5] A. G. Clavier, P. F. Panter, and W. Dite, "Signal-to-Noise-Ratio Improvement in a PCM System," *Proceedings of the I.R.E.*, v. 37, pp. 355–359; April, 1949. Also, *Electrical Communication*, v. 26, pp 257–262; September, 1949.

acteristics can be kept nearly constant and large signal-to-noise ratios maintained even for transmission through a great number of repeaters. It is definitely favorable for radio transmission where such conditions are not encountered. In this respect, pulse-count modulation would seem to be specially advantageous, since regenerative relays can be used in this case, reshaping the pulses at any repeating point. A very small signal increase is sufficient to provide for the required signal-to-noise ratio at the end of the chain.

The more general aspect of the integration of radio-relay links into networks needs also to be considered. Should microwave radio demonstrate its ability to carry a large volume of intercity communications economically, then those transmission methods that are best adapted to radio might be extended to certain other parts of the network, if justified.

7. *Switching*

One interesting feature of pulse transmission in this respect is the possibility of switching channels. A number n of incoming telephone channels, after going through a simultaneous time-sampling operation, can be multiplexed in any desired time sequence by the insertion of suitable delays in the individual channels. Time scanning of the group could connect the individual incoming channels on outgoing terminals in an order determined by the value of the individual channel delay. By changing the value of the time delay introduced in the individual channels, the channel could be effectively switched between incoming and outgoing terminals in any desired manner. A typical example of switching by time displacement is shown in Figure 5.

If a multiplex pulse-transmission system is utilized, the switching operation also can be performed, but the individual pulse channels must first be isolated by a gating operation and passed through individual delay lines, and then reinserted in the multiplex system. At the receiving end, these channels would appear in an order that is determined by the value of the delays inserted.

The delays, which determine interconnection of channels between the ends of the system, may be actual time delays or the equivalent. It is possible to conceive that the delay may be introduced by counting units of time after the arrival of the incoming channel pulses. Such a counting operation can be performed by gas tubes designed for the purpose, the units of time being supplied by a common control element.

Many other methods to attain the same results in this comparatively new field of switching pulse-modulated channels can be visualized, but it is very difficult, at the present time, to evaluate their relative merits.

Pulse, being of the nature of telegraph technique, obviously offers possibilities in the telegraph field; and as switching, as described above, is effected by means that are essentially of a telegraph nature, it can play its part, too.

If the use of the imagination is allowed, rather than established technical and economic facts, it is possible to visualize methods of transmission and switching using pulse modulation, which may permit a useful combination of these two fields that have so far been separate, with an ultimate gain in the complete network.

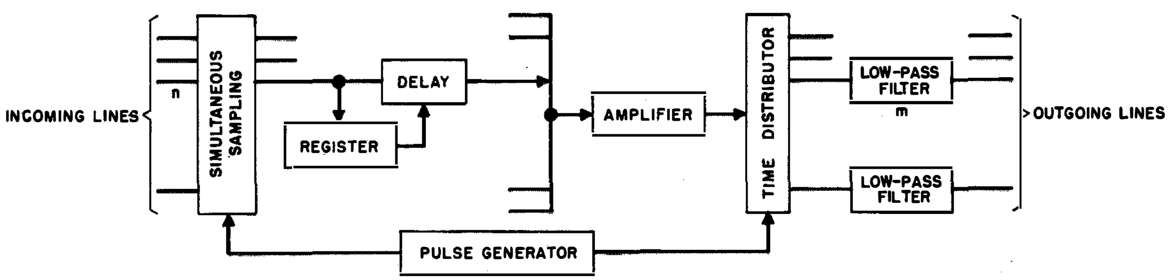

Figure 5—A typical example of switching by time displacement.

Some Relations Between Speed of Indication, Bandwidth, and Signal-to-Random-Noise Ratio in Radio Navigation and Direction Finding*

By H. BUSIGNIES AND M. DISHAL

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RATE of phenomenon change and required speeds of indication are quite slow in many navigational and direction-finding systems, particularly those for long ranges. Therefore, the actual total required electrical bandwidths are also quite narrow, probably never greater than 100 cycles per second or so, and in most cases much less than this. Even with complex wave forms, such small total bandwidths are possible if there can be designed a discontinuous-type bandpass filter having a multiplicity of very narrow pass bands occurring at the steady-state Fourier components of the complex signal; i.e., a "comb" filter. One practical method of producing such a discontinuous pass band is described briefly.

In view of the interest in new modulation schemes that give an output signal-to-noise ratio that is better than the input carrier-to-noise ratio, it is pointed out that all such systems have improvement thresholds, and many navigational systems provide satisfactory information at output signal-to-noise ratios lower than these threshold values. When this is the case, single-sideband and double-sideband amplitude modulation produce the most sensitive systems.

When postdetection bandwidth is very much narrower than predetection bandwidth, many navigational systems will perform satisfactorily even though the carrier-to-noise ratio at the input to the final detector is appreciably less than unity. When this is so, the phenomenon of "apparent demodulation" is encountered. Because it is of practical importance, an analysis, which is useful for most engineering purposes, is performed to find the relation between the open-circuit antenna carrier voltage or available power, the output signal-to-noise ratio $(S/N)_v$ required for satisfactory indicator operation, the percentage modulation m of the carrier, the predetection bandwidth Δf_{IF}, and the postdetection bandwidth Δf_v when a linear final detector is used in a double-sideband amplitude-modulation system.

The above relation depends markedly on whether the quantity $4(\Delta f_{IF}/\Delta f_v)(N/S)_v^2 m^2$ is greater or less than unity, and certain practical cases are investigated to show the dependence of the "required carrier for system operation" on the above quantity.

• • •

1. Symbols

C = root-mean-square carrier voltage in series with the equivalent-generator resistance R_g, which is seen looking back from the receiver input terminals

P_a = available power from equivalent generator of output resistance R_g

N_{IF} = root-mean-square value of thermal-noise voltage appearing in the predetection bandwidth Δf_{IF} and across the input to the linear detector

N_D = root-mean-square value of thermal-noise voltage appearing across the load of the linear detector

N_v = root-mean-square value of thermal-noise voltage appearing in the postdetection bandwidth Δf_v

C_{IF} = root-mean-square value of carrier voltage appearing in the predetection bandwidth Δf_{IF} and across the input to the linear detector

S_D = root-mean-square value of signal; i.e., the envelope of the modulated carrier, appearing across the load of the linear detector

Δf_{IF} = predetection band-pass bandwidth

* Reprinted from *Proceedings of the I.R.E.*, v. 37, pp. 478–488; May, 1949. Presented at Institute of Radio Engineers National Convention, New York, New York, March 4, 1947.

Δf_v = postdetection low-pass bandwidth (Δf_v may also be a band-pass bandwidth if $\Delta f_v \ll \Delta f_{IF}$ and the midfrequency of Δf_v is very much lower than Δf_{IF})

G_{IF} = value of predetection voltage gain

G_D = value of detector voltage gain

G_v = value of postdetection voltage gain

NF = noise figure of receiver

R_g = equivalent-generator output resistance, which is seen looking back from receiver terminals

KT = Boltzman's constant times absolute temperature

m = percentage modulation of the carrier.

Unless otherwise noted, all carrier, noise, and signal currents, voltages, and ratios throughout this paper are root-mean-square values.

2. *Introduction*

That a fundamental relation exists between "required minimum pass bandwidth" and "rate of transmitted information" is well known in the communication field. Also well known is the basic relation between thermal-agitation noise power and the pass bandwidth and, in a more qualitative way, the relation between the pass bandwidth and effect of impulse-type noise on a received signal.

The implications of these relations will be considered with reference to the specific field of radio direction finding and navigation. These considerations are of particular importance in the design of long-range direction-finding and navigational systems. Long range is a relative term; for low-frequency navigational systems, distances of 1500 miles may be considered, whereas for very- and ultra-high-frequency direction finding, line-of-sight distances in the neighborhood of 150 miles are "long" range.

Briefly, we can say that before final detection the root-mean-square thermal-noise voltage is proportional to the square root of bandwidth, and the peak voltage resulting from impulse-type noise is directly proportional to bandwidth. Therefore, the narrowest possible bandwidth should be used to minimize noise, and it is important to know what bandwidth actually is required to pass the transmitted information.

The problem of placing as many channels as possible in a given frequency band is an additional practical reason for using narrow-bandwidth systems.

This paper is not written to recommend specific systems but rather to present a point of view concerning the bandwidth required for the transmission of information concerning direction and distance.

3. *Required Speed of Indication*

With respect to navigational systems, this fundamental point must be realized; when there is no change in the direction of arrival of a signal or in the distance between receiver and transmitter, information involving direction or distance is being conveyed at zero rate and essentially an infinitely small bandwidth is required to indicate this. This is equivalent to saying that whatever wave form is used in a distance- or direction-indicating system, that wave form is in a steady-state condition when there is no change in relative position in the system.

Only when the direction of arrival of a carrier, or the distance between receiver and transmitter is changing, is directional or distance information being received, and the rate at which the direction or distance is changing determines the total bandwidth required at the receiver. This assumes that no communication or other information is to be received at the same time.

In practice, there are actually two "types" of speeds of indication to be considered; one type concerns the rate at which the phenomenon being measured is changing after the phenomenon has already reached its quasi-steady-state condition. This speed of indication is, of course, fixed entirely by the rate of change in the phenomenon; i.e., no matter how large a bandwidth is used, the speed of indication would not increase over its actual occurrence rate.

The other type of speed of indication is involved when the phenomenon is, in effect, "turned on" or "turned off"; e.g., when a direction-finder receiver is suddenly changed in frequency to take bearings on a "new" transmitter. For this case, of course, increasing the bandwidth will increase the speed at which the indicator gives the bearing.

Considering both types of speed of indication, we list below the approximate length of time in which it is desirable that information be obtained in some of the various types of radio direction finders and aids to navigation.

A. Aircraft radio compass—Two or three seconds; must follow the changes of course of the aircraft.

B. Omnidirectional radio range—Two or three seconds; could be much slower as it does not have to change rapidly according to plane course or position, except near the station.

C. Distance-measuring equipment—Two or three seconds would be satisfactory. The present system requires much longer (say, 30 seconds) to provide protection against noise or interference (strobe technique).

D. Long-range navigational systems—Two minutes is considered satisfactory for a fix. The classical loran is slow because of the procedure of operation.

E. Radar—Radar of the rotating-antenna type is definitely limited to the time required to effect a complete turn, 1 to 30 seconds.

4. *Required Bandwidth*

Before considering the specific bandwidths required by the speed of indication noted in Section 3, we would like to review the concept of a discontinuous pass band.

4.1 Concept of Discontinuous Pass Band

With respect to required bandwidth, this fundamental point must be realized: no matter how complex the modulation is on a carrier, the steady-state reception of a complex steady-state signal conveys information at zero rate and requires zero bandwidth; i.e., although a multiplicity of infinitely small bandwidth pass bands may be necessary, essentially no resultant bandwidth is required to receive this steady-state complex signal. Only when the envelope of the modulation on the carrier is changing is information conveyed at any rate and the resultant required bandwidth is fixed by the rate of change of the envelope of the modulation.

As a specific example, consider a steady-state train of modulating pulses; i.e., the time picture before a carrier is modulated.

For the amplitude-versus-time wave form shown in Figure 1(a), it is well known that the envelope of the distribution of amplitude with respect to frequency is that shown in Figure 1(b), and the phase relation between the amplitudes is as shown in Figure 1(c). Mathematically, the envelope of the frequency distribution shown in Figures 1(b) and 1(c) is the Fourier transform of the time distribution shown in Figure 1(a).

When essentially steady-state conditions are reached in the pulse train, the frequency distribution is not continuous, but the amplitudes of Figure 1(b) occur at discrete points on the frequency scale.[1] (The first zero in the envelope

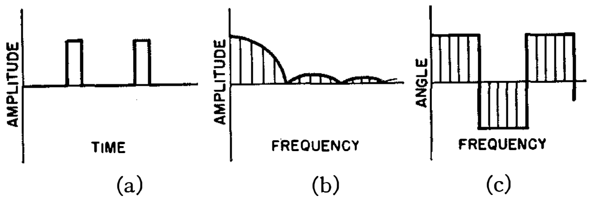


Figure 1—Well-known frequency distribution for a time distribution consisting of pulses.

of Figure 1(b) occurs at a frequency equal to the reciprocal of the pulse width (the other zeros occur at harmonic multiples of this frequency) and the number of discrete frequency lines appearing in each cycle of the envelope is equal to one less than the ratio of the repetition width to the pulse width.)

It is, therefore, possible, and in some cases even practical, to reproduce this steady-state pulse train satisfactorily by combining the outputs of a multiplicity of pass bands that are essentially zero cycles wide and positioned in frequency so as to pass these separate frequency components. The total resultant bandwidth used can thus be extremely small.

Now, if the wave train shown in Figure 1(a) should gradually change to a different steady-state amplitude, a certain bandwidth would be required in each of the separate pass bands to permit these multiple pass bands to follow the change in wave form. The bandwidth required depends on the rate of change of the envelope of the modulating wave form in passing from one quasi-steady state to the next.

A fairly exact indication of the bandwidth required in each pass band to accommodate a given rate of change may be obtained by solving for the response of a band-pass circuit when a sinusoidal driving voltage is suddenly changed in amplitude; i.e., by finding the "rise time" or the "decay time" of the band-pass circuit.

[1] T. E. Shea, "Transmission Networks and Wave Filters," D. Van Nostrand Co., New York, New York, 1929; pp. 417–426.

Making use of the LaPlacian transform or any of the other well-known methods of solution, it can be shown[2,3] that the decay (or rise) time in seconds Δt of a band-pass circuit of Δf is approximately $\Delta t \doteq 1/\Delta f$. It is then approximately true that a band-pass circuit will be able to transmit an envelope if the rate of change of that envelope is much slower than the decay or rise time of the circuit.

Each pass band provided for the Fourier series components must have a width equal to that indicated above and, therefore, the total bandwidth needed is directly proportional to the number of Fourier components required to reproduce the wave form satisfactorily.

It is worth repeating that no bandwidth is required to reproduce a steady-state wave form *no matter how complex:* bandwidth is required only to follow the rate of change of the envelope of a wave form.

Considering the quite slow rates of change in the phenomenon to be measured and the speeds of indication noted in Section 3, most of the navigational aids require total pass bands that are between 1 and 100 cycles wide (for a single-function system measuring only distance or direction) and in many cases are even narrower than this.

[2] E. A. Guillemin, "Communication Networks," John Wiley and Sons, New York, New York; 1935, v. II, pp. 474–506.

[3] M. J. DiToro, "Phase and Amplitude Distortion in Linear Networks," *Proceedings of the I.R.E.*, v. 36, pp. 24–36; January, 1948.

4.2 "Comb-Filter" Circuit

Figure 2 shows a circuit[4] that produces the discontinuous type of pass band described in Section 4.1. The rotational frequency of the brushes determines the frequency location of the pass bands and the time constant R_1C establishes the width of the pass bands ($R_2 \gg R_1$).

When it is impracticable to utilize such a comb filter and continuous pass bands must be used for navigational systems, the slow rate of phenomenon change should indicate the use of simple wave forms having quite long periods.

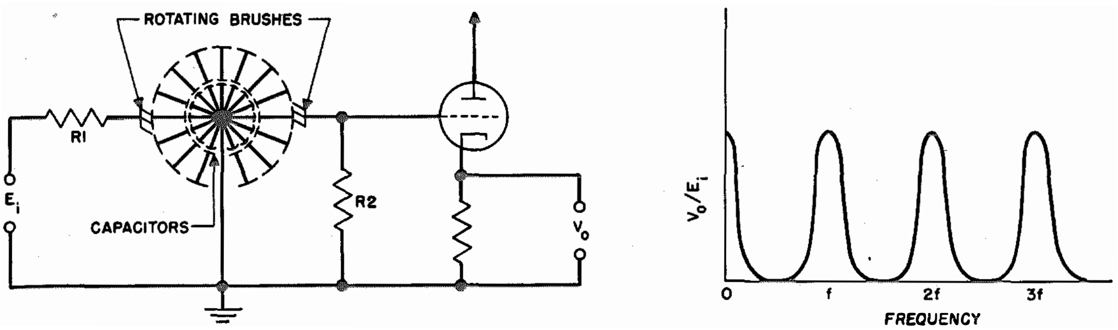
5. Some Relations Between Bandwidth and Thermal Signal-to-Noise Ratios

5.1 Concerning the "Best" Type of Modulation

Discussions have appeared recently of the concept that the output signal-to-noise ratio of an information-transmitting system can be increased by increasing the receiver bandwidth and using the proper kind of modulation.[5] Consequently, some readers may feel that the concepts of this paper apply only to one type of modulation; i.e., amplitude modulation. This limitation is not correct as the following brief discussion should indicate.

[4] Developed by G. R. Clark of Federal Telecommunication Laboratories, Nutley, New Jersey.

[5] D. G. F., "Bandwidth vs. Noise in Communication Systems," *Electronics*, v. 21, pp. 72–75; January, 1948.



(a) (b)

Figure 2—(a) Electromechanical circuit that produces a "comb filter" type of frequency response. (b) type of frequency response produced. The mechanical rotational frequency f fixes the location of the pass bands and the time constant $R_1C(R_2 \gg R_1)$ fixes the width of each pass band.

The fact must be realized that in a great many, perhaps even in a majority, of cases satisfactory navigational information can be obtained at quite low output signal-to-noise ratios $(S/N)_v$; i.e., approximately 2:1, say.

Now in all modulation schemes where the output signal-to-noise ratio from the final detector is better than the input signal-to-noise ratio there is always a predetection carrier-to-noise ratio $(C/N)_{IF}$ *improvement threshold*, which must be passed before the improvement is obtained. Of course, the wider one makes the predetection bandwidth to obtain a larger improvement factor, the greater will be the number of microvolts required to reach this improvement threshold.

The input required to reach the improvement threshold in wide-band systems is always larger than that needed to produce a 2:1 output signal-to-noise ratio in a single-sideband amplitude-modulation system, which uses the narrowest possible band.

Thus, when satisfactory results can be obtained with output signal-to-noise ratios approximately equal to the predetection carrier-to-noise ratio improvement threshold for the various wide-band systems, the most sensitive system (i.e., the one that will operate satisfactorily with the smallest transmitted power) will employ single-sideband amplitude modulation; double-sideband amplitude modulation will be next best.

5.2 Relations Between Received Carrier, Output Signal-to-Noise Ratio, and Pre- and Postdetection Bandwidths for Double-Sideband Amplitude Modulation with a Linear Final Detector

When a linear detector is used in a given double-sideband amplitude-modulation system (and for a number of practical reasons this is usually the case), the output signal-to-noise ratio will be found to be directly proportional to the strength of the applied modulated carrier only down to a certain carrier level; below this value, the output signal-to-noise ratio approaches proportionality to the square of the applied modulated-carrier voltage.

This unfortunate effect is due to the phenomenon of "apparent demodulation," which occurs when the carrier-to-noise ratio at the input to a linear detector is in the neighborhood of unity or is less than unity. Ragazzini[6] has considered this phenomenon both analytically and experimentally for the case where the carrier-to-noise ratio at the input to the linear detector does not drop much below unity.

In the previous section, it was noted that satisfactory navigational information can be obtained at output signal-to-noise ratios as low as 2:1. Therefore, when the predetection bandwidth is much wider than the postdetection bandwidth (as is often the case), an output signal-to-noise ratio of 2:1 means that the predetection carrier-to-noise ratio can be appreciably less than 1:1. It thus seemed worth while to extend Ragazzini's work by investigating the phenomenon of apparent demodulation for the case of predetection carrier-to-noise ratios materially less than 1:1. Since it is of practical importance, we will consider the relations between carrier level C, percentage modulation m, predetection bandwidth Δf_{IF}, postdetection bandwidth Δf_v, and output signal-to-noise ratio $(S/N)_v$ in the postdetection bandwidth. This relation will be derived with satisfactory accuracy for most engineering applications.

Since the complete, exact analysis, for *any* predetection carrier-to-noise ratio, of the case where noise plus a modulated carrier is applied to a linear detector leads to quite formidable mathematical manipulations, we will obtain our desired engineering relations by a combination of analysis and experiment.

We will first follow the random noise through the system of Figure 3 to obtain the resulting noise N_v in the final postdetection bandwidth Δf_v (for the sake of brevity, we will refer to this as the video-frequency bandwidth); the effect of the signal on the noise will, of course, be considered. Next, we will follow the signal through the system to find the resulting signal S_v in the final video-frequency bandwidth Δf_v; the effect of the noise on the signal will, of course, be considered. And finally, our desired output signal-to-noise ratio $(S/N)_v$ will be given by the ratio of the above signal to the above noise.

It should be noted that to simplify reasoning, the response of the diode load is made at least

[6] J. R. Ragazzini, "Effect of Fluctuation Voltages on the Linear Detector," *Proceedings of the I.R.E.*, v. 30, pp. 277–288; June, 1942.

equal to or greater than the full intermediate-frequency bandwidth (i.e., not half that bandwidth). When video-frequency narrowing is obtained after the diode load, it is isolated from that load so as not to impair the frequency response of the diode load. This is normal good design practice insofar as negative-peak clipping, sideband cutting, and similar factors are concerned.

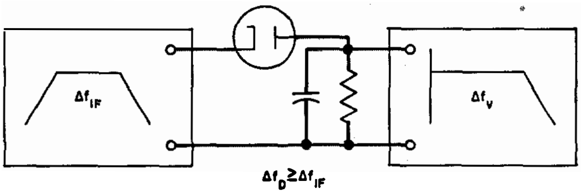


Figure 3—The three parts of the receiver in which the relation between signal and noise is to be considered.

5.2.1 Resulting Noise in Postdetection Bandwidth

First we must consider the relation between the noise at the input to the detector and the resulting noise across the diode load.

Landon[7] has shown that, with no carrier present and neglecting diode efficiency or gain, the alternating-current noise voltage output N_D from a perfect linear detector is equal to 0.655 of the noise oscillations N_{IF} entering the diode.

Bennett[8] has shown that, as carrier is added to the noise, the alternating-current noise voltage across the diode load increases by a maximum of approximately 1.6 times as the carrier-to-noise ratio becomes very large. Actually, at a predetection carrier-to-noise ratio of about 3:1, 90 percent of this increase has already occurred. Stated in another way, when a strong carrier plus noise is fed to the input of a linear detector, the alternating-current noise voltage across the diode load N_D equals the input noise voltage N_{IF}, i.e., 1.6×0.655. Modulation on the received carrier produces an additional small percentage increase in the alternating-current noise across the diode load.

In view of the above quite-small (for most purposes) variation in noise across the diode load (i.e., from 0.655 N_{IF} to 1.0 N_{IF}), we will make the engineering approximation that

$$N_D \doteq 0.7 N_{IF} G_D. \qquad (1)$$

It is well known that the noise in the intermediate-frequency pass band at the input to the diode detector can be expressed in terms of the intermediate-frequency band width and noise figure NF of the receiver in the following manner.

$$N_{IF} = G_{IF}(4R_g KT\Delta f_{IF} NF)^{1/2}, \qquad (2)$$

where R_g is the equivalent-generator output resistance seen looking back from the receiver input terminals, and G_{IF} is the voltage gain of the receiver with reference to the open-circuit voltage of the equivalent generator; i.e., G_{IF} is the ratio of the root-mean-square voltage across the output terminals of the last intermediate-frequency transformer to the root-mean-square open-circuit voltage of the equivalent generator.

Equation (2) can now be written as

$$N_D = 0.7(4R_g KT\Delta f_{IF} NF)^{1/2} G_{IF} G_D. \qquad (3)$$

We must next consider the relation between this noise across the diode load and the resulting noise in the final video-frequency bandwidth.

This relation is complicated by the fact that at input carrier-to-noise ratios below unity, the frequency spectrum of the noise across the diode load is approximately triangular, falling linearly to zero at a video frequency equal to the full intermediate-frequency bandwidth; whereas at input carrier-to-noise ratios above unity, the frequency spectrum of the noise across the diode load is approximately rectangular cutting off at a video frequency equal to one-half the intermediate-frequency bandwidth.[9] By the following approximations, we will obtain an expression for the desired relation that is satisfactory for the great majority of engineering applications.

First: For the case of carrier-to-noise ratios below unity, we will assume that the frequency

[7] V. D. Landon, "Distribution of Amplitude with Time in Fluctuation Noise," *Proceedings of the I.R.E.*, v. 29, pp. 50–55; February, 1941; Discussion, v. 30, pp. 425–429; September, 1942.

[8] W. R. Bennett, "Response of a Linear Rectifier to Signal and Noise," *Journal of the Acoustical Society of America*, v. 15, pp. 164–170; January, 1944.

[9] S. O. Rice, "Mathematical Analysis of Random Noise," *Bell System Technical Journal*, v. 24, p. 148; January, 1945.

spectrum of the noise across the diode load is essentially triangular. Figures 4(a) and 4(b) then give the relation to be considered. The noise power across the diode load is given by A_1 and

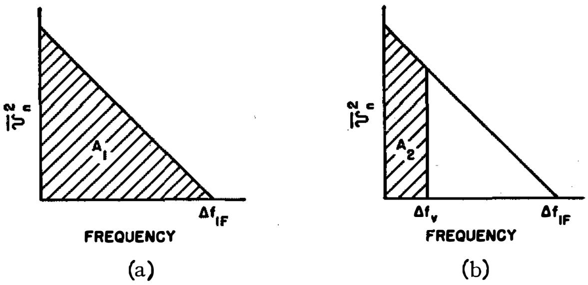


Figure 4—The relation between the noise power across the detector load and the noise power in the postdetection bandwidth at very low predetection carrier-to-noise ratios.

the noise power in the video-frequency bandwidth is given by A_2. The square root of the ratio of these two values is our desired relation and from simple geometric considerations can be written

$$\frac{N_v}{N_D}=\left(\frac{\Delta f_v}{\Delta f_{IF}}\right)^{1/2}\left(2-\frac{\Delta f_v}{\Delta f_{IF}}\right)^{1/2}G_v. \qquad (4)$$

It should be realized, of course, that the maximum meaningful value of $(\Delta f_v/\Delta f_{IF})$ in (4) is unity, because after the video-frequency bandwidth becomes wider than the intermediate-frequency bandwidth there is no change in the video-frequency noise output. It is evident that the second factor of (4) varies from a maximum of 1.4 to a minimum of 1.0. For most engineering purposes, this is not a large variation and, because in many practical cases the video-frequency bandwidth is much smaller than the intermediate-frequency bandwidth, we will use (4a) as an approximation for (4).

$$\frac{N_v}{N_D}\doteq 1.4\left(\frac{\Delta f_v}{\Delta f_{IF}}\right)^{1/2}G_v. \qquad (4a)$$

Second: Now let us consider the case of predetection carrier-to-noise ratios above unity. Here the frequency spectrum of the noise across the diode load is essentially rectangular and Figure 5 gives the relation to be considered.

As in Figure 4, the desired ratio of N_v/N_D is given by the square root of the ratio of the areas A_1 and A_2 of Figures 5(a) and 5(b). By simple geometry, the ratio is

$$\frac{N_v}{N_D}=1.4\left(\frac{\Delta f_v}{\Delta f_{IF}}\right)^{1/2}G_v. \qquad (5)$$

We see that (5) for the case of predetection carrier-to-noise ratios above unity is the same as our approximation (4a) for ratios below unity. Thus, for most engineering purposes we can use (4a) or (5) to give the desired relation between the noise in the video-frequency pass band and the noise across the diode load.

Combining (3) and (5), we have our final desired noise equation,

$$N_v\doteq(4R_gKTNF)^{1/2}\Delta f_v^{1/2}G_{IF}G_DG_v. \qquad (6)$$

5.2.2 Resulting Sine-Wave Signal in the Postdetection Bandwidth

As previously mentioned, Ragazzini[6] has used an analytical approach to this problem that is valid (because of the use of a convergent series) down to predetection carrier-to-noise ratios of approximately unity. Ragazzini also considers the additional relatively small effects of modulation compression and modulation distortion, which we will neglect.

Figure 6 is an experimentally obtained graph giving the desired relation between the resulting sine-wave signal across the diode load and the input modulated-carrier level as the carrier is varied above and below the value that gives unity carrier-to-noise ratio at the input to the second detector. (The apparatus used to obtain Figure 6 is described in Section 8.) It will be noted that the abscissas and ordinates are ex-

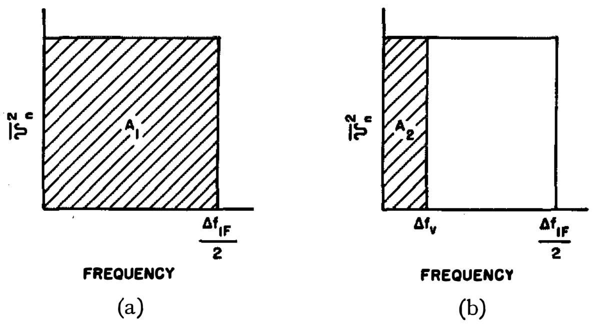


Figure 5—The relation between noise power across the detector load and noise power in the postdetection bandwidth at high predetection carrier-to-noise ratios.

pressed in terms of carrier level $C_{1:1}$, which makes the carrier equal to the noise.

The circles on the graph are the experimentally obtained points and the solid-line curve is a plot of (7)

$$S_v = G_{IF}G_D \frac{mC}{\left[1+\left(\frac{N_{IF}}{C_{IF}}\right)^2\right]^{1/2}}. \tag{7}$$

(Of course, both sides of (7) were divided by $G_{IF}G_D mC_{1:1}$ for plotting Figure 6.) For most purposes, (7) is an excellent representation of the experimental data, and we will, therefore, consider that (7) adequately describes the phenomenon of apparent demodulation. (Equation (7) was obtained by a guess based on the shape of the experimental curve of Figure 6 and on (18) of footnote reference 6.)

Finally, since we are assuming that the video-frequency pass bandwidth is wide enough to accommodate the desired signal, we have the fact that

$$S_v = G_v S_D \tag{8}$$

and combining (7) and (8) and making use of (2), we obtain

$$S_v = \frac{mC}{\left[1+\frac{4R_gKT\Delta f_{IF}NF}{C^2}\right]^{1/2}} G_{IF}G_DG_v. \tag{9}$$

5.2.3 Value of Modulated Carrier Required to Produce a Given Signal-to-Noise Ratio in the Postdetection Bandwidth

Dividing (9) by (6), we obtain

$$\left(\frac{S}{N}\right)_v \doteq \frac{mC}{(4R_gKT\Delta f_vNF)^{1/2}} \cdot \left[\frac{1}{1+\frac{4R_gKT\Delta f_{IF}NF}{C^2}}\right]^{1/2}. \tag{10}$$

For any given value of modulated carrier and pre- and postdetection bandwidths, we can calculate the approximate signal-to-noise ratio in the final pass band from (10).

Solving (10) for the required carrier value C, we obtain

$$C \doteq (4R_gKT\Delta f_vNF)^{1/2}\left(\frac{1}{m}\right)\left(\frac{N}{S}\right)_v \cdot \left\{\frac{1}{2}+\frac{1}{2}\left[1+4\left(\frac{\Delta f_{IF}}{\Delta f_v}\right)\left(\frac{N}{S}\right)_v^2 m^2\right]^{1/2}\right\}^{1/2} \tag{11}$$

or, in terms of available power from the equivalent-signal generator, we have

$$P_a = \frac{C^2}{4R_g} = KT\Delta f_vNF\left(\frac{1}{m}\right)^2\left(\frac{S}{N}\right)_v^2 \cdot \left\{\frac{1}{2}+\frac{1}{2}\left[1+4\left(\frac{\Delta f_{IF}}{\Delta f_v}\right)\left(\frac{N}{S}\right)_v^2 m^2\right]^{1/2}\right\}. \tag{11a}$$

Equations (11) and (11a) are the important equations of this section of the paper; for any combination of variables in the equation, we can obtain the required carrier to make the system operate.

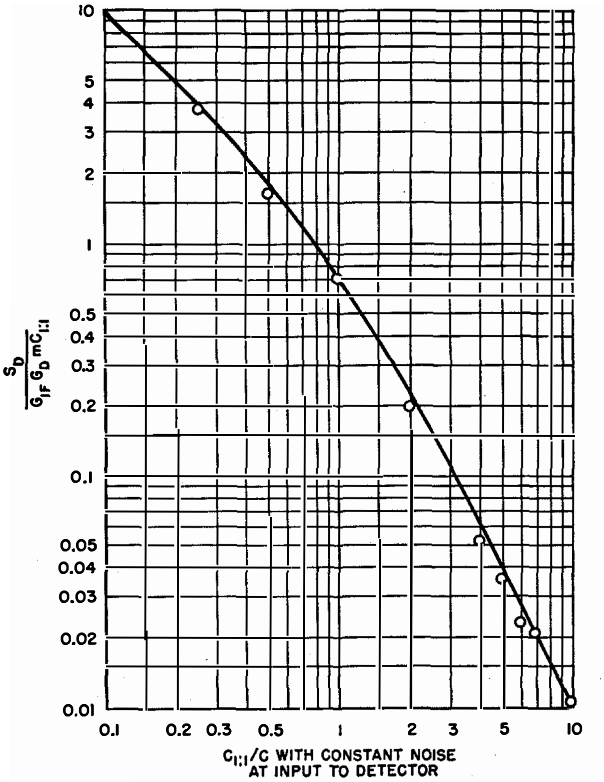


Figure 6—The relation, for a linear detector, between the resulting signal that appears across the detector load, and the input carrier-to-noise ratio. $G_{IF}G_DmC_{1:1}$ is the signal actually fed to the input to the detector at a 1:1 carrier-to-noise ratio. At unity on the abscissa scale, the root-mean-square values of carrier and noise are equal.

There are two conditions worth considering wherein (11) reduces to quite-simple useful forms. These conditions are set by the quantity

$$4\left(\frac{\Delta f_{IF}}{\Delta f_v}\right)\left(\frac{N}{S}\right)_v^2 m^2$$

being either much greater or much less than unity.

6. Practical Applications

6.1 Required Carrier When $4(\Delta f_{IF}/\Delta f_v) \times (N/S)_v^2 m^2 \gg 1$

We will first give one example of a parctical system in which the condition $4(\Delta f_{IF}/\Delta f_v) \times (N/S)_v^2 m^2 \gg 1$ is satisfied. A long-range navigational system is to be designed. The indicator reading time can be as long as 10 seconds; therefore, the effective postdetection bandwidth should be of the order of 0.1 cycle. Because of transmitter and local-oscillator frequency stabilities, the predetection bandwidth will have to be of the order of 20 cycles. Therefore, $\Delta f_{IF}/\Delta f_v$ will always be very much larger than unity. The indicator to be used will give satisfactory readings at a video-frequency signal-to-noise ratio of approximately 2:1. The percentage modulation on the carrier in the final intermediate-frequency circuit will be as close as possible to 100 percent. In this system, $4(\Delta f_{IF}/\Delta f_v) \times (N/S)_v^2 m^2$ is much larger than unity. Probably the most important single condition is the fact that because of practical limitations the predetection bandwidth must be very much larger than the postdetection bandwidth. This particular condition is true of a number of navigational systems.

When $4(\Delta f_{IF}/\Delta f_v)(N/S)_v^2 m^2 > 1$, (11) reduces to

$$C \doteq (4R_g KTNF)^{1/2}(\Delta f_{IF}\Delta f_v)^{1/4}\left(\frac{S}{N}\right)_v^{1/2}\left(\frac{1}{m}\right)^{1/2} \quad (12)$$

or in terms of available power

$$P_a = \frac{C^2}{4R_g} = KTNF(\Delta f_{IF}\Delta f_v)^{1/2}\left(\frac{S}{N}\right)_v \frac{1}{m}. \quad (12a)$$

Because these conditions (for which (12) and (12a) are true) apply to many practical systems, these equations are quite useful insofar as design considerations are concerned. As an example, we note that the required carrier for system operation is proportional to the *fourth root* of the intermediate-frequency bandwidth. In the light of this result, let us again consider the example given at the beginning of this section. Suppose the frequency-stability problem mentioned there would be materially helped by doubling the intermediate-frequency bandwidth (from 20 to 40 cycles); we see from (12) that the required carrier for system operation would be increased by only 1.19 times. In view of this small loss in system sensitivity, it might be well worth while increasing the intermediate-frequency bandwidth.

6.2 Required Carrier When $4(\Delta f_{IF}/\Delta f_v) \times (N/S)_v^2 m^2 \dot{\lessgtr} 1$

This condition is usually satisfied by those navigational systems where it is not necessary to make the predetection bandwidth much wider than approximately twice the required postdetection bandwidth (as should be done in double-sideband amplitude-modulation systems). In practice, this usually means that the system requires a rather large postdetection bandwidth. A fast-reading-time automatic direction finder using a cathode-ray-tube indicator is an example of this type of system.

For this case, (11) and (11a) reduce to (13) and (13a).

$$C = (4R_g KTNF)^{1/2}\Delta f_v^{1/2}\left(\frac{S}{N}\right)_v \frac{1}{m}, \quad (13)$$

$$P_a = KTNF\Delta f_v\left(\frac{S}{N}\right)_v^2\left(\frac{1}{m}\right)^2. \quad (13a)$$

When the navigational system satisfies the conditions of this subsection, we see that the required carrier for system operation is proportional to the square root of the video-frequency bandwidth; and any possible reduction in the video-frequency bandwidth will increase the sensitivity of the system in accordance with (13).

6.3 Adapting a Wide-Band Receiver Design to a Narrow-Band Navigational System

The title of this section describes a situation that does arise in practice. In a specific case, for example, it was requested that a proposed ultra-high-frequency receiver having an intermediate-frequency pass band of 100 kilocycles and a video-frequency pass band of 50 kilocycles be used in a direction-finding system of a type that required a postdetection bandwidth of only 1 kilocycle. Four practical questions arise.

First: What will be the maximum range of the direction finder if this wide-band receiver is used; i.e., what will be the required value of the open-circuit carrier voltage C_1 supplied by the antenna system?

Second: By what factor would we improve the range of the direction-finding system if we redesigned the *post*detection circuits of the receiver so that instead of a 50-kilocycle bandwidth the minimum allowable postdetection bandwidth of 1 kilocycle was obtained; i.e., what will be the required value of open-circuit carrier voltage C_2?

Third: By what factor would we improve the range of the direction-finding system if we redesigned the *pre*detection circuits of the receiver so that instead of a 100-kilocycle bandwidth the minimum allowable predetection bandwidth of 2 kilocycles was obtained; i.e., what will be the required value of open-circuit carrier voltage C_3?

Fourth: How much more improvement will be obtained if the *pre*detection redesign of the third question is carried out as compared to the *post*detection redesign of the second question; i.e., what is the ratio of the required carriers C_2 and C_3?

The important characteristic of the above questions is that a *large* bandwidth reduction ratio will result in either case.

First: Equation (11) answers the first question, and since satisfactory bearing information can be obtained at a 2:1 signal-to-noise ratio in the video-frequency bandwidth, the approximation of Section 6.2 and (13) applies so that we can write for the required carrier value C_1

$$C_1 = 0.7 \Delta f_{IF1}{}^{1/2}\left(\frac{S}{N}\right)_v \frac{1}{m}(4R_gKTNF)^{1/2}. \qquad (14)$$

Second: For the required carrier value in the second question, (11) applies again, and since the approximation of Section 6.1 and (12) applies in this second question, we can write for the new required value of the carrier C_2,

$$C_2 = (\Delta f_{IF1}\Delta f_{v2})^{1/4}\left(\frac{S}{N}\right)_v^{1/2} \times \left(\frac{1}{m}\right)^{1/2}(4R_gKTNF)^{1/2}. \qquad (15)$$

The ratio of the two carrier values C_2 and C_1 will answer question two

$$\frac{C_1}{C_2} = \frac{0.7}{m^{1/2}}\left(\frac{\Delta f_{IF1}}{\Delta f_{v2}}\right)^{1/4}\left(\frac{S}{N}\right)_v^{1/2}. \qquad (16)$$

Note the important fact that for the conditions of this practical example, a redesign of the postdetection circuits makes the ratio of the two required carriers proportional to the *fourth root* of the ratio of the original intermediate- to new video-frequency bandwidths; i.e., even if a large amount of bandwidth reduction is accomplished by redesigning the postdetection circuits, we will obtain only a rather small improvement in range. However, it should be realized that, in many cases, even the small resulting improvement may be ample payment for the redesign effort required.

Third: with reference to the third question, (11) in its approximation form of (13) applies; i.e., the conditions of Section 6.2 will effectively be satisfied, and so (17) gives the carrier C_3 that will be required for system operation if we redesign the predetection circuits of the receiver.

$$C_3 = \frac{0.7}{m}\Delta f_{IF2}{}^{1/2}\left(\frac{S}{N}\right)_v (4R_gKTNF)^{1/2}. \qquad (17)$$

The ratio of the two carrier values C_3 and C_1 will answer question three.

$$\frac{C_1}{C_3} = \left(\frac{\Delta f_{IF1}}{\Delta f_{IF2}}\right)^{1/2}. \qquad (18)$$

We see that, if we redesign the predetection circuits of the wide-band receiver, the ratio of the two carriers C_1 and C_3 required for system operation will be equal to the square root of the ratio of the original to the new intermediate-frequency bandwidths.

Fourth: The answer to the fourth question is given by the ratio of C_2 and C_3.

$$\frac{C_2}{C_3} = \frac{\left(\dfrac{\Delta f_{IF1}}{\Delta f_{IF2}}\right)^{1/2} 1.4(m)^{1/2}}{\left(\dfrac{\Delta f_{IF1}}{\Delta f_{v2}}\right)^{1/4}\left(\dfrac{S}{N}\right)_v^{1/2}}. \qquad (19)$$

Examination of (19) shows that if, in the contemplated redesign, the new postdetection and predetection bandwidths will be about the same width (much narrower than the original bandwidths) then markedly more improvement in range will be obtained if we redesign the predetection circuits of the receiver rather than the postdetection circuits.

It must be realized that the redesign of the predetection circuits of a receiver is usually a problem of a much greater magnitude than that of redesigning the postdetection circuits. Thus, it is necessary for the designer to weigh this greater complexity against the increased range obtained by a predetection redesign.

To illustrate the above points, Figure 7 shows three photographs of a typical pointer-type indication obtained on the cathode-ray-tube indicator of a rotating-loop (or equivalent) direction finder in the 300-megacycle region.

Figure 7(a) was obtained with an ultra-high-frequency receiver having a predetector bandwidth of 100 kilocycles and a postdetector bandwidth of 50 kilocycles. The carrier level C_1 was set so that the output signal-to-noise ratio in the video-frequency circuits is approximately 3:1. The carrier was modulated 100 percent downward by the direction-finding system.

To obtain Figure 7(b), the postdetection circuits of this receiver were redesigned so as to have a low-pass bandwidth of 1 kilocycle and, as indicated, it was then possible to use a new lower carrier level C_2 approximately $\frac{1}{3}$ the original value C_1 and still obtain a video-frequency signal-to-noise ratio approximately the same as that in Figure 7(a). In view of the fact that the 1-kilocycle video-frequency pass band of Figure 7(b) had a very gradual cutoff, (12) seems to describe satisfactorily these experimental results for most engineering purposes.

To obtain Figure 7(c), the predetection circuits of the receiver were redesigned so as to have a bandpass of 2 kilocycles. It was then possible to use a new lower carrier level C_3 approximately $\frac{1}{7}$ the original value C_1 and still obtain a video-frequency signal-to-noise ratio approximately the same as in Figure 7(a). It will be noted that (13) is in satisfactory agreement with the experimental results.

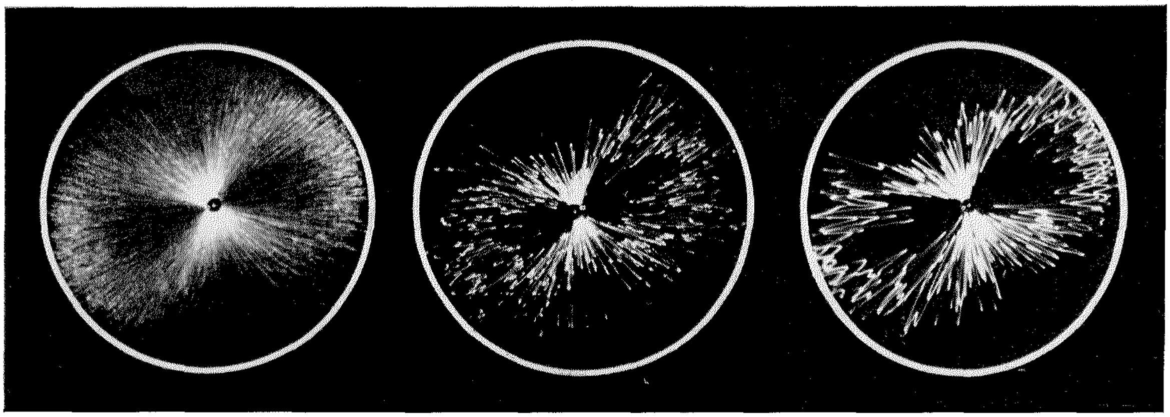
Thus, in this redesign, bandwidth narrowing before final detection produced a system that was more than twice as sensitive as that obtained when the same amount of bandwidth narrowing was accomplished after final detection.

In a practical application of the above reasoning, the range of an ultra-high-frequency direction finder was increased from 30 miles to more than 100 miles by decreasing the intermediate-frequency bandwidth until it was as small as could be used.

7. *Postdetection Bandwidth*

It is important to realize that the postdetection bandwidth Δf_v, continually referred to in this paper, means the bandwidth between the detector and the final indication device. (In some high-speed indicators, the bandwidth response of the human eye may set the postdetection bandwidth.)

In many cases, it is possible to design indicators that make use of seemingly nonlinear elements; e.g., rectifiers; and in these cases, the linear concept of a pass bandwidth may not seem to apply rigorously. However, if the *reading time*



(a) (b) (c)

Figure 7—Pointer-type pattern on a cathode-ray-tube indicator of a rotating-figure-eight direction finder at approximately 3:1 output signal-to-noise ratio. (a) Wide-band receiver, $\Delta f_{IF} = 100$ kilocycles and $\Delta f_v = 50$ kilocycles. (Required carrier was C_1.) (b) Postdetection circuits of receiver redesigned so that $\Delta f_v = 1$ kilocycle. (Required carrier was $C_2 \doteq C_1/3$.) (c) Predetection circuits of receiver redesigned so that $\Delta f_{IF} = 2$ kilocycles. (Required carrier was $C_3 \doteq C_1/7$.)

of the indicator is determined under good postdetection signal-to-noise conditions, then for most engineering purposes the postdetection bandwidth, if of the band-pass type, is quite accurately equal to the inverse of the reading time or to the inverse of twice the bandwidth if of the low-pass type. *Reading time* may be defined in the following manner: A constant bearing with good output signal-to-noise conditions is suddenly applied to the system; the reading time is the time that elapses between the instant when the resulting indication has changed from 5 to 95 percent of the total amount by which it will change.

In many automatic direction finders, a receiving antenna pattern is rotated. It is possible to use this known rotational frequency to produce a number of different types of *synchronous indicators*. These synchronous indicators have the common characteristic that a known reference frequency can be used so that the *average* produced indication is proportional to only that component of receiver output that is of exactly the same frequency as the reference frequency.[10] Note the importance of the word *average* in the above sentence. An important part of a synchronous indicator is a narrow-bandwidth circuit (either band pass or low pass depending on the type of synchronous indicator used) whose function is to remove the instantaneous fluctuations from the bearing indication leaving only the average indication. Since many of these synchronous indicators make use of nonlinear elements, the measurement or analysis of the *reading time* of the indicator as described previously is perhaps the best way to find the effective postdetection bandwidth.

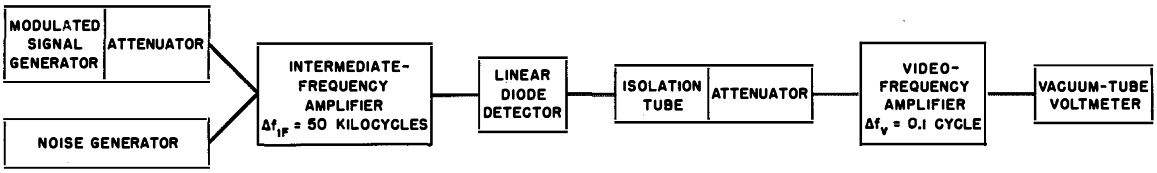


Figure 8—Block diagram of the apparatus used for experimentally determining the law of "apparent demodulation" versus input carrier-to-noise ratio that occurs in a linear diode at input carrier-to-noise ratios down to 1:10.

The quantity S/N_v is an important term in the equations of this paper. It should be realized that it is possible to express this ratio in the postdetection bandwidth Δf_v in terms of the magnitude of fluctuation that will be seen about the true or average bearing. If the noise produces an error-function type of bearing fluctuation (which is true in many practical cases) then it is practical to express numerically the bearing fluctuation in terms of the statistical quantity, the standard deviation of the bearing from the average bearing.

8. *Appendix: Experimental Determination of the Relation Between the Postdetection Signal-to-Noise Ratio and the Predetection Carrier-to-Noise Ratio When a Linear Detector is Used With Double-Sideband Amplitude Modulation*

A block diagram of the electrical apparatus is given in Figure 8.

8.1 Signal Generator and Modulation

The signal generator and attenuator were high-quality standard laboratory instruments. Modulation at 80 percent was produced by an alternator driven by a 15-cycle synchronous motor, which was run from the power lines. The band-

[10] Insofar as operation at low predetection carrier-to-noise ratios is concerned, this exact knowledge of the law of effective rotation of the antenna pattern seems to be one of the fundamental advantages that direction finders of the "receiving type" have over direction finders of the "transmitting type."

In the transmitting case (e.g., an omnirange system), it is always necessary to send to the receiver "synchronizing information," and in many cases, the inability to detect this synchronizing information, rather than inability to detect the directional information, sets the limit on the operating range of the system.

width of the postdetection band-pass filter requires that the 15-cycle modulating frequency have a short-time frequency stability better than approximately 0.02 cycle. By using the power lines to obtain the modulating frequency, the necessary short-time frequency-stability requirement was satisfied.

8.2 Noise Generator

The noise generator consisted of a high-gain double-superheterodyne receiver with a local oscillator and mixer added to its final intermediate-frequency amplifier so that the resulting thermal noise was produced at the desired mid-frequency of 1.5 megacycles. By using the multiple superheterodyne receiver, it was possible to obtain large thermal noise output with no observable regeneration caused by undesired feedback.

8.3 Final Intermediate-Frequency Amplifier

The final intermediate-frequency amplifier from which the linear detector operated and which set the predetection bandwidth of 50 kilocycles, used five cascaded critical-shape-coupled double-tuned circuits and showed no regeneration effects whatsoever. Of course, the signal generator and noise generator, which operate into this amplifier, were carefully decoupled from each other so that any adjustments on one generator had no effect on the output of the other.

8.4 Linear Detector

The linear detector consisted of two sections in parallel of a *6AL5* double diode. The frequency response of the diode load used was approximately 150 kilocycles so that the total triangular noise spectrum was fully reproduced. The alternating- to direct-current impedance ratio of the total diode load for the modulation frequency of 15 cycles was unity (for all practical purposes) so that the 80-percent modulation used could be handled with no observable distortion. The power-output capabilities of the last intermediate-frequency stage driving the linear detector was such that 50 volts peak-to-peak of detected signal could be obtained from the 80-percent modulated carrier before appreciable distortion could be seen. In the experiment, the diode was operated at an essentially constant output level of 5 direct volts resulting from carrier plus noise. This output voltage was sufficiently below the overload point and sufficiently above the contact-potential voltage so that errors due to both effects were negligible. With reference to the linearity of the detector, no attempt was made to measure the departure from a constant value of the slope of the curve giving output direct current versus input carrier; it is standard practice, however, to consider a diode detector to be satisfactorily linear above an output of approximately 2 direct volts, the upper limit being set by the stage driving the detector. Since this experiment dealt mainly with predetector signal-to-noise ratios below unity, it was possible to keep the operating level of the detector essentially constant; i.e., signal plus noise produced approximately 5 direct volts output from the detector.

8.5 Postdetector Narrow-Band Filter

Since an expected signal amplitude variation of at least 100:1 was to occur across the diode load, the attenuator shown in Figure 8 preceding the narrow-band postdetection filter was used so that this filter and the following rectifier voltmeter could be operated at a constant level; thus the problem of the amplitude linearity of the vacuum tubes in the filter did not have to be considered. Since the circuits following this attenuator then operate at a low level, it is necessary to be sure that hum and audio-frequency noise have negligible effect.

The required bandwidth for the postdetection filter was fixed by the predetection bandwidth of 50 kilocycles, by the fact that it was desired to obtain readings at predetection carrier-to-noise ratios as low as 1:10, and by the fact that to read the output meter accurately it was necessary to have a reasonably good signal-to-noise ratio, say 5:1, in the postdetection bandwidth at this 1:10 input carrier-to-noise ratio. To calculate this required postdetection bandwidth, we combine (1) and (4a) and obtain

$$N_v \doteq \left(\frac{\Delta f_v}{\Delta f_{IF}}\right)^{1/2} N_{IF} G_D G_v. \tag{20}$$

Next, combine (7) and (8) to obtain

$$S_v = \frac{mC_{IF}}{\left[1+\left(\frac{N_{IF}}{C_{IF}}\right)^2\right]^{1/2}} G_D G_v. \tag{21}$$

The ratio of (20) and (21) is

$$\left(\frac{S}{N}\right)_v = \left(\frac{\Delta f_{IF}}{\Delta f_v}\right)^{1/2} \frac{m\left(\frac{C}{N}\right)_{IF}}{\left[1+\left(\frac{N}{C}\right)^2_{IF}\right]^{1/2}} \tag{22}$$

and, finally, for the case we are interested in where $(N_{IF}/C_{IF})>1$, we obtain

$$\frac{\Delta f_{IF}}{\Delta f_v} \doteq \frac{\left(\frac{S}{N}\right)^2_v}{m^2\left(\frac{C}{N}\right)^4_{IF}}. \tag{23}$$

For the set of conditions previously mentioned, we see that the ratio of predetection to postdetection bandwidth must be 2.5×10^5; thus we require a 0.2-cycle wide postdetection bandwidth centered at the modulation frequency of 15 cycles. In the actual experiment, a 3-decibel bandwidth of approximately 0.1 cycle was used. A rough check of the signal-to-noise ratio in this postdetection bandwidth at a predetection carrier-to-noise ratio of 1:10 showed that (23) gives satisfactory engineering accuracy.

This bandwidth was obtained by cascading two band-pass amplifier stages. Each stage consisted of a feedback-amplifier chain using the well-known parallel-T null network in the feedback path. Each feedback-amplifier chain had a gain of approximately 350 at 15 cycles, thus giving a resonant-circuit Q of approximately 85. Each chain was direct-current coupled throughout so that there was no low frequency at which the feedback could become regenerative; and by the simple expedient of dropping the high-frequency response of *one* of the stages in the chain the magnitude of the $\mu\beta$ gain was made less than unity at the high frequency at which the phase shift about the feedback chain was 360 degrees. Since there is 100-percent negative feedback for direct current in each chain, direct-current drift troubles due to the direct-current coupling are not encountered.

It is important that the parallel-T null network be so accurately aligned that its loss at the null frequency is greater than the gain of the feedback amplifier in which it is used, and it is necessary to devise a rigorous alignment procedure. It is also important to realize that the frequency-selective characteristics of this type of effective band-pass circuit are limited by the bandwidth of the feedback amplifier and by the signal-handling capabilities of the tubes in the chain. It will be remembered that, in the experiment, the postdetection circuits must reject noise-frequency components up to 50 kilocycles. Rather than design the feedback amplifiers for this pass band, they were designed to handle a few thousand cycles and a few sections of RC low-pass filters preceded the feedback filter so that no frequency components above a few thousand cycles ever reached the feedback band-pass circuits.

8.6 Setting for 1:1 Predetection Carrier-to-Noise Ratio

Landon in his discussion with Norton[7] showed that with only noise present, the average output A of a perfect linear detector; i.e., the direct-current value read on a long-time-constant direct-current voltmeter, is $A_N=1.252\ N$, where N is the root-mean-square value of the intermediate-frequency noise oscillations. We know that when carrier alone is present, the average output A_c of a perfect linear detector is $A_c=1.414\ C$, where C is the root-mean-square value of the intermediate-frequency carrier oscillations. Therefore, if we have the ability temporarily to cut off the carrier input and thus read the direct-current output A_N due to noise only, and then temporarily cut off the noise input and read the direct-current output A_c due to carrier only, the ratio of the readings will be given by (24).

$$\frac{A_N}{A_c} = 0.886\frac{N}{C}. \tag{24}$$

Thus, for our desired predetection carrier-to-noise ratio of 1:1, the ratio of the readings was made to be 0.886. This ratio was checked before each reading to make sure the noise generator was supplying a stable and constant output.

This setting was also checked in another way: Bennett[8] has shown that when carrier is added to the noise at the input to a linear detector, the resulting direct-current output of the detector is approximately 1.4 times the direct-current output produced by noise alone. The frequency response of the diode load must be equal to or greater than the full intermediate-frequency bandwidth, and there must be enough noise (e.g., 2 volts) to ensure that the diode is in its linear region. This ratio was found to check very satisfactorily with the ratio given by (24).

The graph of Figure 6 was then obtained by varying the signal-generator attenuator from this 1:1 predetection carrier-to-noise condition and finding the corresponding required variation in the audio-frequency attenuator to maintain a constant output from the postdetection filter.

Recent Telecommunication Development

Reference Data for Radio Engineers, Third Edition

A THIRD EDITION of Reference Data for Radio Engineers has been published by Federal Telephone and Radio Corporation. This 672-page volume is twice as large as the previous edition. It not only contains information unavailable for the first two editions, but the earlier material has been expanded wherever experience revealed a definite need. Over one hundred thousand copies of the first two editions were sold since 1943.

The book was compiled by engineers of Federal Telecommunication Laboratories and of other associate companies of the International Telephone and Telegraph Corporation. Most of these engineers are regular contributors to *Electrical Communication.*

While many of the chapters of the second edition have been retained, they have been revised and new material added. In some cases, the names of the chapters have been changed to comply with more advanced terminology or to cover increased scope. For example, the former "Room Acoustics" chapter now includes a more comprehensive treatment of acoustics and its name has been changed to "Electroacoustics." In the same sense, the material previously under "Audio and Radio Design" was expanded into five chapters; namely, "Components", "Fundamentals of Networks", "Selective Circuits", "Filter Networks", and "Attenuators."

Among the new chapters added are: "Frequency Data", including comprehensive material from the 1947 Atlantic City Conference; "Bridges and Impedance Measurements"; "Modulation"; "Fourier Waveform Analysis"; "Radar Fundamentals"; "Broadcasting"; "Servomechanisms"; and "Maxwell's Equations."

The chapter on "Radio Propagation and Noise", of 26 pages in the second edition, has been expanded into two chapters covering 62 pages.

Material previously called "General Information" has been expanded into three chapters entitled: "Units, Constants, and Conversion Factors"; "Properties of Materials"; and "Miscellaneous Data."

"Mathematical Formulas" take 32 pages more than the 12 pages in the second edition.

Although it contains twice as many pages as its predecessor, the third edition is only slightly thicker and but a few ounces heavier through the use of thinner paper.

Copies of Reference Data for Radio Engineers, third edition, may be ordered from Federal Telephone and Radio Corporation, Publication Department, 67 Broad Street, New York 4, N.Y., at $3.75 a copy or, in lots of 12 or more copies to a single address, at $3.00 each, postpaid.

Variations of Telephone Traffic

By F. W. RABE
Bell Telephone Manufacturing Company, Antwerp, Belgium

THE number of circuits required per group for an automatic telephone exchange depends on the average traffic density during a number of busy hours.

To determine the number of circuits per group for a certain average traffic density, under full-availability conditions, increasing use is made of Erlang's well-known probability equation for lost calls.

The theoretical assumptions underlying Erlang's equation B for lost calls are the following:

A. Each call (attempt) is made independently of time and of the number of calls in progress. If n represents the average call density or the average number of calls offered in the unit of time to a group of circuits, the probability that a call will be made during an infinitely small time dt will be equal to ndt.

B. Holding times may be constant or variable. The average holding time of the successful calls will be represented by h.

C. The average values of n and h should be based on a large number of calls originated in a long time T_1, during which the traffic conditions that influence the averages should remain constant.

D. All calls should have access to all circuits of the group (full-availability condition). Traffic fulfilling conditions A, B, and C will be referred to as "stationary traffic."

The product of h and n or the average number of calls offered during the average holding time of the successful calls is the traffic value included in Erlang's equation.

In case of small losses, hn may be replaced by y, where y represents the average traffic density or the average number of circuits simultaneously engaged, in the group under consideration, during a long time T_1.

When measuring the average traffic density during short time intervals, y_1 values will be obtained and will vary around the average y value.

To determine the busy hour, average traffic densities will have to be measured during short time periods (for instance, quarters of an hour) during the busiest period of the day. A large number of daily busy hours will form a long time comparable with the theoretical long time T_1. During this time, the traffic should so far as possible be stationary, and the number of circuits required is determined by Erlang's equation B.

To check whether this condition is really fulfilled, the actual variations of y_1 around y should be compared with the theoretical variations of y_1 for stationary traffic having the same average y.

This check can be made in a rather easy way by comparing the standard deviations for the two cases.

$$[\sum(y-y_1)^2P(y_1)]^{1/2}.$$

In this formula, $P(y_1)$ represents the probability for an average y_1 during a short period in the theoretical case and for the practical case the proportion of short periods having an average y_1.

In the following two sections, equations for the standard deviations for the theoretical case, for constant as well as for varying holding times, will be derived. The application of the equations will be demonstrated in Section 3.

1. Variations of Stationary Traffic—Constant Holding Times

The probability that a certain number of calls n_1 originate in a short time interval t_1 is given by the second form of Poisson's equation:

$$P(n_1t_1)=\frac{e^{-nt_1}(nt_1)^{n_1}}{n_1!}. \qquad (1)$$

The average time interval between two consecutive calls will be $1/n$ for a large number of calls. For a small number of calls n_1, this average time interval will vary and will be equal to α/n, where α may have all values from 0 to ∞.

Instead of counting the number of calls in each equal period t_1, we may also measure the time intervals t_1 for an equal number of calls n_1. Each time interval starts then at the moment the first call is originated and terminates at the moment call n_1+1, i.e., the first call of the next period t_1, is originated. The first call has a definite place at the beginning of the period (probability ndt_1) whereas the other n_1-1 calls are distributed at random over the period t_1 according to (1).

The time t_1 will be equal to $(n_1/n)\alpha$ so that $nt_1=n_1\alpha$ and $ndt_1=n_1d\alpha$.

The probability of finding n_1 consecutive calls with an average time interval α/n will be

$$P(n_1\alpha)=\frac{e^{-n_1\alpha}(n_1\alpha)^{n_1-1}n_1d\alpha}{(n_1-1)!}. \qquad (2)$$

To facilitate calculations, two assumptions will be made:

A. The number of circuits is infinity or the loss is very small and negligible.

B. Each time unit t_1 is complete in itself. This means that if the holding time of a call falls partly outside t_1, it returns with the remaining part of the holding time to the beginning of that period. If t_1 is short, the same call may return several times.

In this way the average traffic density y_1 during the time t_1 will always be equal to

$$\frac{n_1h}{t_1}=\frac{n_1nh}{nt_1}=\frac{n_1y}{nt_1}.$$

It will be evident that, especially for small values of t_1, the average traffic density y_1 found in this way will be too high.

The standard deviation for the average traffic densities during equal time intervals t_1 may now be found by applying (1).

$$\sigma_c(t_1)=\left[\sum_{n_1=0}^{n_1=\infty}\frac{(y_1-y)^2e^{-nt_1}(nt_1)^{n_1}}{n_1!}\right]^{1/2},$$

$$\sigma_c(t_1)=y\left[\sum_{n_1=0}^{n_1=\infty}\left(\frac{n_1^2}{n^2t_1^2}-\frac{2n_1}{nt_1}+1\right)\frac{e^{-nt_1}(nt_1)^{n_1}}{n_1!}\right]^{1/2}.$$

When replacing n_1^2 by $n_1(n_1-1)+n_1$, it will be easy to find the standard deviation $\sigma_c(t_1)$ for constant holding times.

$$\sigma_c(t_1)=y\left(\frac{1}{nt_1}\right)^{1/2}. \qquad (3)$$

When t_1 is long, the standard deviation will be small and for $t_1=\infty$, $\sigma_c(t_1)=0$.

On the other hand, when t_1 is short, the standard deviation according to (3) will be large and for $t_1=0$, $\sigma_c(t_1)=\infty$. This latter case cannot be correct and is due to assumption B (t_1 complete in itself).

The correct standard deviation for momentary values of y_1, has been calculated.[1]

$$\sigma(0)=y^{1/2}=y\left(\frac{1}{y}\right)^{1/2}. \qquad (4)$$

This equation may be derived from the first form of Poisson's equation, giving the probability for momentary values of y_1, for which case y_1 will be an integer.

$$P(y_1)=\frac{e^{-y}y^{y_1}}{y_1!}. \qquad (5)$$

The standard deviation $\sigma(0)$ will then be

$$\sigma(0)=\left[\sum_{y_1=0}^{y_1=\infty}(y-y_1)^2\frac{e^{-y}y^{y_1}}{y_1!}\right]^{1/2}.$$

From this, (4) is obtained by replacing y_1^2 by $y_1(y_1-1)+y_1$.

As Poisson's (5) is valid for constant as well as for variable holding times, (4) also will be valid for all holding times.

In Figure 1, the solid-line curve A shows the standard deviation for average traffic densities during short time intervals t_1 as a function of t_1/h and for constant holding times according to (3).

$$\sigma_c(t_1)=y\left(\frac{1}{nt_1}\right)^{1/2}=\left(\frac{y}{nt_1}\right)^{1/2}\cdot(y)^{1/2}=\left(\frac{h}{t_1}\right)^{1/2}\cdot y^{1/2}.$$

For $t_1/h=0$, the standard deviation must be equal to $(y)^{1/2}$. Curve A' shows approximately how curve A should be modified to obtain the standard deviation for y_1 for small values of t_1/h.

There are two reasons for the deviation of curve A' from A. The first, which is due to assumption B, has been mentioned already. It indicates that the standard deviation corresponding to (3) is too large, especially for small values of t_1/h. The second reason is that the number of conversations at the beginning of the period t_1 continuing from the preceding period need not be the same as the number of conversations existing at the end of period t_1. This difference makes the standard deviation corresponding to (3) too small. For long time intervals t_1, the effect is relatively small; it is also small for short periods as the number of conversations at the start and end of short time intervals t_1 are not

[1] L. von Bortkewitsch, "Das Gesetz der kleinen Zahlen" (Law of Small Numbers), Leipzig; 1898.

independent of each other. There will, therefore, be a time t_1 for which the influence is maximum.

As we are only interested in periods having a length of at least 15 minutes when $h=2$ to 3 minutes, no attempt will be made to find the correct mathematical expression for curve A'. We have found the standard deviation for y_1 by considering equal time periods t_1 in (3). We may, however, also consider an equal number of calls n_1. By (2), the standard deviation $\sigma_c(n_1)$ will become

$$\sigma_c(n_1)=\left[\int_{\alpha=0}^{\alpha=\infty}\frac{(y-y_1)^2\,e^{-n_1\alpha}(n_1\alpha)^{n_1-1}n_1d\alpha}{(n_1-1)!}\right]^{1/2},$$

$$\sigma_c(n_1)=y\left[\frac{1}{(n_1-1)!}\int_{\alpha=0}^{\alpha=\infty}\left(\frac{1}{\alpha^2}-\frac{2}{\alpha}+1\right)e^{-n_1\alpha}(n_1\alpha)^{n_1-1}n_1d\alpha\right]^{1/2},$$

$$\sigma_c(n_1)=y\left\{\frac{1}{(n_1-1)!}\left[n_1^2\Gamma(n_1-2)-2n_1\Gamma(n_1-1)+\Gamma(n_1)\right]\right\}^{1/2},$$

$$\sigma_c(n_1)=y\left\{\frac{1}{(n_1-1)!}\left[n_1^2(n_1-3)!-2n_1(n_1-2)!+(n_1-1)!\right]\right\}^{1/2}.$$

Hence

$$\sigma_c(n_1)=y\left[\frac{n_1+2}{(n_1-1)(n_1-2)}\right]^{1/2}. \quad (6)$$

For $n_1=1$ or 2, the standard deviation equals ∞, due to the fact that for one or two calls the influence of short time intervals t_1 becomes evident. For a large number of calls n_1, we find approximately

$$\sigma_c(n_1)\approx y\left(\frac{1}{n_1}\right)^{1/2}. \quad (7)$$

Equation (7) is of less importance than (3) as it is less practical to measure the average traffic densities for equal numbers of calls n_1 than for equal time intervals t_1.

2. *Variations of Stationary Traffic—Varying Holding Times*

We assume that the probability for the holding time of a call being terminated during an infinitely small time dt is dt/h and that it is independent of how long the call has been in progress at the moment considered.

Consider a fictitious arrangement in which the calls are placed one after the other in such a way that the holding time of call $i+1$ starts exactly at the moment the holding time of call i terminates. In this way, a large number of calls will combine to form a long time T_2, which is quite different from the long time T_1 noted before. As a matter of fact, $T_2=T_1nh=T_1y$.

Arbitrarily, let us take a short time t_2 out of T_2 and try to find the probability that there will be n_2 calls (starts and terminations of conversations coinciding) during this time.

This probability will be determined by the second form of Poisson's equation, so that a formula similar to (1) is obtained.

$$P(n_2t_2)=e^{-t_2/h}\left(\frac{t_2}{h}\right)^{n_2}\frac{1}{n_2!}. \quad (8)$$

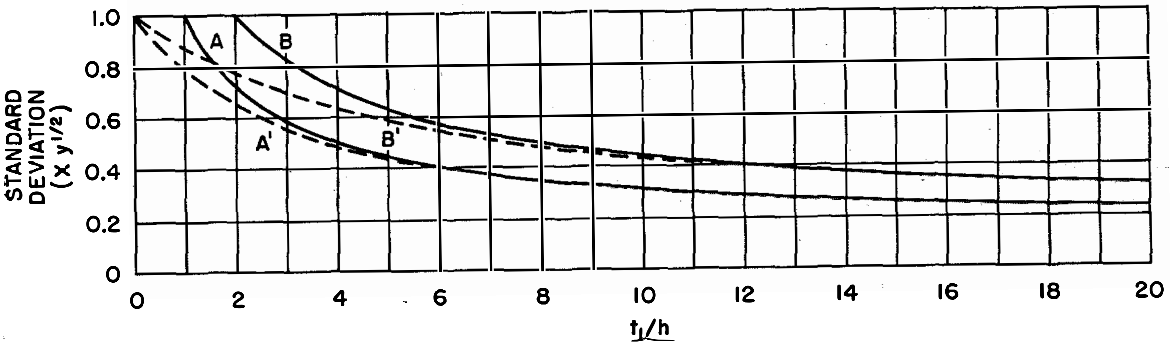


Figure 1—Standard deviation plotted against t_1/h.
Curve $A=(h/t_1)^{1/2}y^{1/2}$ (constant holding time), Curve $B=(2h/t_1)^{1/2}y^{1/2}$ (varying holding time).

On the other hand, we can measure the period t_2 comprising exactly n_2 calls and ask for the probability of n_2 calls having an average holding time equal to βh. The time t_2 will be equal to $n_2\beta h$ so that $t_2/h = n_2\beta$.

We will obtain an equation similar to (2).

$$P(n_2\beta) = \frac{e^{-n_2\beta}(n_2\beta)^{n_2-1}n_2d\beta}{(n_2-1)!}. \tag{9}$$

The same assumptions, A and B, will be made as for constant holding time.

$$\sigma_v(t_1) = y\left[\sum_{n_1=0}^{n_1=\infty}\frac{e^{-nt_1}(nt_1)^{n_1}}{n_1!(n_1-1)!}\int_0^\infty e^{-nt_1\gamma}(nt_1\gamma)^{n_1-1}(1-\gamma)^2 nt_1 d\gamma\right]^{1/2},$$

$$\sigma_v(t_1) = y\left\{\sum_{n_1=0}^{n_1=\infty}\frac{e^{-nt_1}(nt_1)^{n_1}}{n_1!(n_1-1)!}\left[\Gamma(n_1)-\frac{2}{nt_1}\Gamma(n_1+1)+\frac{1}{n^2t_1^2}\Gamma(n_1+2)\right]\right\}^{1/2},$$

$$\sigma_v(t_1) = y\left\{\sum_{n_1=0}^{n_1=\infty}\frac{e^{-nt_1}(nt_1)^{n_1}}{n_1!}\left[1-\frac{2n_1}{nt_1}+\frac{(n_1+1)n_1}{n^2t_1^2}\right]\right\}^{1/2}.$$

It will be evident that when taking $n_2 = n_1$,

$$y_1 = \frac{t_2}{t_1} = \frac{n_1\beta h}{\frac{n_1}{n}\alpha} = \frac{nh\beta}{\alpha} = y\gamma,$$

when $\gamma = \beta/\alpha$.

We have now to consider two variations, viz., the variation of the average time interval between consecutive calls and the variation in average holding time.

We will determine now the probability of measuring an average traffic density y_1 during a short time interval t_1. The probability that there will be n_1 calls during this time t_1 will be given by (1). For an average traffic density $=y_1$, the time t_2 should be equal to y_1t_1. The probability of finding an average holding time βh for $n_2 = n_1$ calls is given by (9). In this equation, however, $n_2\beta$ is replaced by $n_1\beta = n_1\gamma\alpha = nt_1\gamma$ and $n_2d\beta$ is replaced by $nt_1d\gamma$.

The probability $P(\gamma t_1 n_1)$ of finding an average traffic density $y_1 = \gamma y$ during a time t_1 for n_1 calls will be found by multiplying (1) by (9), which gives

$$P(\gamma t_1 n_1) = \frac{e^{-nt_1}(nt_1)^{n_1}}{n_1!}\cdot\frac{e^{-nt_1\gamma}(nt_1\gamma)^{n_1-1}nt_1d\gamma}{(n_1-1)!}.$$

To find the total probability $P(\gamma t_1)$, we must consider $n_1 = 0, 1, 2,$ up to ∞ and add all these probabilities together so that

$$P(\gamma t_1) = \sum_{n_1=0}^{n_1=\infty} P(\gamma t_1 n_1)$$

or

$$P(\gamma t_1) = \sum_{n_1=0}^{n_1=\infty}\frac{e^{-nt_1(1+\gamma)}\gamma^{n_1-1}(nt_1)^{2n_1}d\gamma}{n_1!(n_1-1)!}. \tag{10}$$

The standard deviation $\sigma_v(t_1)$ will be equal to

$$[\sum(y-y_1)^2P(\gamma t_1)]^{1/2}$$

or

Hence

$$\sigma_v(t_1) = y\left(\frac{2}{nt_1}\right)^{1/2}. \tag{11}$$

This equation is similar to (3) for constant holding times. As a matter of fact, we have found that $\sigma_v(t_1) = (2)^{1/2}\sigma_c(t_1)$. Curve B shows the standard deviation for average traffic densities during short time intervals t_1 as a function of t_1/h according to (11). Curve B' shows approximately how curve B should be modified for small values of t_1/h.

We may replace (3) and (11) by a single equation,

$$\sigma(t_1) = y\left(\frac{\delta}{nt_1}\right)^{1/2}. \tag{12}$$

In this formula, $\delta = 1$ for constant holding times whereas $\delta = 2$ for varying holding times, fulfilling the condition stipulated at the beginning of this section. For mixed holding times, δ will have a value between 1 and 2.

Finally, the probability may be determined for an average traffic density $y_1 = \gamma y$ for n_1 consecutive calls. For this purpose, multiply (2) by (9).

$$P(n_1\alpha)P(n_2\beta) = \frac{e^{-n_1\alpha}(n_1\alpha)^{n_1-1}n_1d\alpha}{(n_1-1)!}\cdot\frac{e^{-n_2\beta}(n_2\beta)^{n_2-1}n_2d\beta}{(n_2-1)!}.$$

We take $n_1 = n_2$, whereas $\gamma = \beta/\alpha$. β and α can be varied by multiplying them by the same value, so that γ remains constant. We replace α by β/γ and assume that β varies from 0 to ∞. The probability $P(n_1\gamma)$ of finding an average traffic density γy for n_1 calls will be

$$P(n_1\gamma) = \int_{\beta=0}^{\beta=\infty} \frac{e^{-n_1\beta[(1/\gamma)+1]}(n_1\beta)^{2n_1-1}}{(n_1-1)!(n_1-1)!} \cdot \frac{n_1}{\gamma^{n_1+1}} d\gamma d\beta,$$

$$P(n_1\gamma) = \frac{\gamma^{n_1-1} d\gamma}{(n_1-1)!(n_1-1)!(\gamma+1)^{2n_1}} \Gamma(2n_1).$$

Hence

$$P(n_1\gamma) = \frac{(2n_1-1)!}{(n_1-1)!(n_1-1)!} \cdot \frac{\gamma^{n_1-1}}{(\gamma+1)^{2n_1}} d\gamma. \quad (13)$$

The standard deviation of the average traffic density γy for an equal number of calls n_1 will be

$$\sigma_v(n_1) = [\sum (y-y_1)^2 P(n_1\gamma)]^{1/2},$$

$$\sigma_v(n_1) = y\left[\frac{(2n_1-1)!}{(n_1-1)!(n_1-1)!}\int_0^\infty \frac{(1-\gamma)^2\gamma^{n_1-1}}{(\gamma+1)^{2n_1}} d\gamma\right]^{1/2},$$

$$\sigma_v(n_1) = y\left\{\frac{(2n_1-1)!}{(n_1-1)!(n_1-1)!}[B(n_1, n_1) - 2B(n_1+1, n_1-1) + B(n_1+2, n_1-2)]\right\}^{1/2},$$

$$\sigma_v(n_1) = y\left[\frac{2n_1+2}{(n_1-1)(n_1-2)}\right]^{1/2}. \quad (14)$$

Also for this equation, if $n_1 = 1$ or 2, the standard deviation will become ∞. For a large number of calls n_1, the standard deviation will approximate the value

$$\sigma_v(n_1) \approx y\left(\frac{2}{n_1}\right)^{1/2}. \quad (15)$$

When comparing (7) and (15), we find that

$$\sigma_v(n_1) \approx (2)^{1/2}\sigma_c(n_1).$$

3. *Application of (12)*

The average traffic densities during equal time intervals for various groups of circuits can be measured simultaneously with automatic traffic-recording equipment. To analyse these traffic figures and determine if, and how far, the variations of these average traffic densities exceed the variations of corresponding stationary traffic as covered by Erlang's probability equation, we must also count the number of calls or measure the average holding time. Further, we must investigate whether the holding times are constant or variable or whether both types are mixed.

To illustrate the way in which (12) may be used, a numerical example will be given.

We assume that average traffic intensities for one group of circuits during 8 consecutive periods of ¼ hour each have been recorded as shown in Table 1.

It is assumed that the total number of calls during 2 hours = 234 or an average of 29.2 per quarter and the holding times for 90 percent of the calls are variable and for 10 percent, constant.

Only 8 observations are available to determine the average traffic density y, and these are not sufficient to calculate the exact value of y.

We must, therefore, take the most probable value for y, which is the arithmetical mean, $y = 3.75$, for both the actual and theoretical cases. The sum of $(y-y_1)^2$ will be 5.67 so that the standard deviation of actual average traffic densities will be $(5.67/8)^{1/2} = 0.84$.

As 10 percent of the holding times are constant, we take $\delta = 1.9$. The theoretical standard deviation for stationary traffic will, therefore, be $3.75\ (1.9/29.2)^{1/2} = 0.96$ when applying (12). As

TABLE 1

AVERAGE TRAFFIC DENSITY FOR EIGHT CONSECUTIVE QUARTER-HOUR PERIODS

Quarter Hour	Average Traffic Density $= y_1$	$y - y_1$	$(y-y_1)^2$	Number of Calls
1	3.3	0.45	0.20	25
2	3.0	0.75	0.57	22
3	4.2	−0.45	0.20	30
4	3.2	0.55	0.30	28
5	5.5	−1.75	3.06	40
6	4.5	−0.75	0.57	30
7	3.0	0.75	0.57	29
8	3.3	0.45	0.20	30
Total	30.0	0	5.67	234
Average	$3.75 = y$	0	0.71	29.2

The traffic density is a pure figure without dimension. Recently, the name "Erlang" has been given to the traffic density unit. One Erlang = 30 equated busy-hour calls (EBHC).

the standard deviation for actual traffic (0.84) is smaller than the theoretical standard deviation (0.96), the traffic may be considered as being stationary during the 2 hours.

A deviation $(y_1 - y)$ greater than twice the standard deviation will occur only 2 to 4 times out of 100 cases for stationary traffic and a deviation greater than 3 times the standard deviation will occur only 2 to 4 times out of 1000 cases.

If all values in Table 1 had been 10 times greater, we would have found a standard deviation of actual average traffic densities $= 10 \times 0.84 = 8.4$. The corresponding theoretical standard deviation for stationary traffic, however, would have been $37.5(1.9/292)^{1/2} = 0.96\ (10)^{1/2} = 3.04$. For this case, it would not have been possible to consider the traffic as being stationary during the 2 hours.

When the time intervals t_1 are shorter than 15 minutes and the average holding time is 2 minutes, (12) may no longer be used. The approximate curves A' and B' may for such a case (e.g. $t_1 = 5$ minutes) be helpful and will permit approximate values to be found for the standard deviation.

Equation (12) may also be useful for determining the average busy hour of an exchange. The average traffic density of the average busy hour should preferably be found by taking the average of about 300 traffic observations made during the course of one year.

When calculating the standard deviation for these 300 values, it may be found that it is larger than that which corresponds to (12). In such a case, a sufficient number of the lowest values should be omitted to make the standard deviation for the remaining values correspond to the theoretical standard deviation. The latter will increase, as the average of the remaining values becomes greater.

Recent Telecommunication Development

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The booklet is available from Federal Telephone and Radio Corporation, 900 Passaic Avenue, East Newark, New Jersey, at twenty-five cents per copy.

Design Equations for Servomechanisms*

By BENJAMIN PARZEN

Federal Telecommunication Laboratories, Incorporated, Nutley, New Jersey

A SERVO SYSTEM is a combination of elements for the control of a source of power in which the output of the system or some function of the output is fed back for comparison with the input and the difference between these quantities is used in controlling the power.[1]

In this definition, the term "input" includes broadly any auxiliary reference source with which the output sample is compared to obtain an error quantity for correction purposes. Such a reference source may be, but is not necessarily, a pure source of the characteristic under control. For instance, whereas an automatic-frequency-control system usually compares the output frequency with an auxiliary oscillator of high frequency stability, an automatic-gain-control amplifier utilizes the amplitude of the output signal to produce a bias voltage that controls the gain of the amplifier. There is, in this case, no pure source of gain with which the gain of the amplifier is compared.

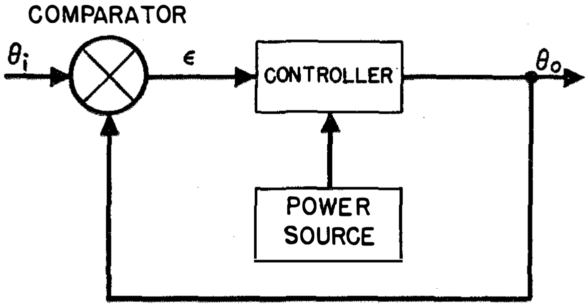


Figure 1—Example of simple servo system.

Servo systems may include electric, hydraulic, pneumatic, and other forms of power. Systems that involve mechanical motion are called servomechanisms and only such systems are treated in this paper.

1. Basic System Elements

The basic elements of a servomechanism are shown in Figure 1 and include an input or reference quantity θ_i, an output quantity θ_o, a mixer or comparator that subtracts θ_o from θ_i to yield an error quantity $\epsilon = \theta_i - \theta_o$, and a controller, which so regulates the flow of power from the power source that ϵ tends toward zero. The controller may include amplifiers, motors, and other devices.

2. Classification

Servomechanisms may be classified in accordance with their use or with their motive or control characteristics. Some of those classified by their uses include remote control, power amplification, indicating instruments, and computers. Among those classified in accordance with their motive characteristics are hydraulic servos, thyratron servos, Ward-Leonard controls, amplidyne controls, two-phase alternating-current servos, mechanical-torque amplifiers, and pneumatic servos. Under control characteristics will be found the relay-type servo, in which the full power of the motor is applied as soon as the error is large enough to operate a relay; definite-correction servo, where the power of the motor is controlled in finite steps at definite time intervals; and continuous-control servos, in which the power of the motor is continuously controlled by some function of the error. Only the continuous type of servomechanism is treated in this paper.

3. Fundamental Quantities for Linear Lumped-Constant Servo Systems

$f(t)$ = function of time (1)

$F(p)$ = Laplace transform of $f(t)$ (2)

θ_i = input quantity (3)

* Reprinted from the third edition of "Reference Data for Radio Engineers" published by Federal Telephone and Radio Corporation.

[1] James, Nichols, and Phillips, "Theory of Servomechanisms," (Radiation Laboratories Series, Number 25) McGraw-Hill Book Company, New York, New York, 1947.

$$\theta_o = \text{output quantity} \tag{4}$$

$$\epsilon = \text{error quantity} = \theta_i - \theta_o \tag{5}$$

$Y(p)$ = loop transfer function

$= \dfrac{\theta_o(p)}{\epsilon(p)} = \dfrac{|KQ_m(p)}{p^s P_n(p)}$, where $m < n$ and s is an integer. $|K$ is defined in (7). Q_m and P_n are polynomials of degree m and n, of which the coefficient of zero power of p is taken as unity. (6)

$$|K = \text{loop gain} = \lim_{p \to 0} p^s Y(p) \tag{7}$$

$$Y_o(p) = \text{over-all transfer function} = \frac{\theta_o(p)}{\theta_i(p)} = \frac{Y(p)}{1+Y(p)} = |K_o \frac{S_m(p)}{R_n(p)}, \tag{8}$$

where S_m, R_n are polynomials similar to Q_m and P_n in (6) above

$$Y_i(p) = \text{error-input transfer function} = \frac{\epsilon(p)}{\theta_i(p)} = \frac{1}{1+Y(p)} = \frac{p^s P_n(p)}{1+|KQ_m(p)} \tag{9}$$

$$f_{ss} = \text{steady-state quantity} = \underset{t\to\infty}{f(t)} = \lim_{p \to 0} pF(p). \tag{10}$$

When $s=1$ in (6) the system is termed a zero-displacement-error system, since from equations (9) and (10), $\epsilon_{ss}=0$ when $\theta_i(t)$ is a step displacement. Similarly, when $s=2$, the system is termed a zero-velocity-error system since $\epsilon_{ss}=0$ when $\theta_i(t)$ is a step velocity. Obviously a zero-velocity-error system is also a zero-displacement-error system.

4. *Positioning Type Servomechanisms*

The fundamental quantities described above are applicable to all classifications of continuous-control servo system. The remaining material in this paper applies to positioning systems using electronic and electromechanical devices. Other servo systems can be treated in exactly analogous fashions.

A typical positioning servo is shown in Figure 2. For this system:

$$Y(p) = \frac{\theta_o(p)}{\epsilon(p)} = \frac{k_1 Y_A(p) Y_m(p) U(p)}{1 + Y_m(p) U(p) V(p)}, \tag{11}$$

$$Y_o(p) = \frac{\theta_o(p)}{\theta_i(p)} = \frac{k_1 Y_A(p) Y_m(p) U(p)}{1 + k_1 Y_A(p) Y_m(p) U(p) + Y_m(p) U(p) V(p)}, \tag{12}$$

$$Y_i(p) = \frac{\epsilon(p)}{\theta_i(p)} = \frac{1 + Y_m(p) U(p) V(p)}{1 + k_1 Y_A(p) Y_m(p) U(p) + Y_m(p) U(p) V(p)}. \tag{13}$$

In Figure 2, comparator 1 is an error-measuring system that converts the difference between θ_i and θ_o into error voltage e, where $e = k_1\epsilon$. k_1 is usually a real constant. Examples of error-measuring systems are shown in Figure 3.

Mixer 2 in Figure 2 is a circuit arrangement that subtracts E_c from E_a to yield a voltage $e_1 = E_a - E_c$. $U(p)$ represents the motor and load characteristics. It includes the motor gearing and all inertias and forces imposed by the load. Quantities and relations making up and describing $U(p)$ are described by (14) to (39).

4.1 $U(p)$

In the following, subscript m refers to motor, l refers to load, and o refers to combined motor and load:

$$\theta = \text{angular position in radians} \tag{14}$$

$$\Omega = \text{angular velocity in radians per second} = d\theta/dt \tag{15}$$

$$M_m = \text{motor-developed torque in foot-pounds} \tag{16}$$

$$J_m = \text{motor inertia in slug-feet}^2 \tag{17}$$

$$E_m = \text{impressed volts} \tag{18}$$

$$k_t = \text{motor stalled-torque constant in foot-pounds per volt} = (\Delta M_m/\Delta E_m)_{\Omega_m} \tag{19}$$

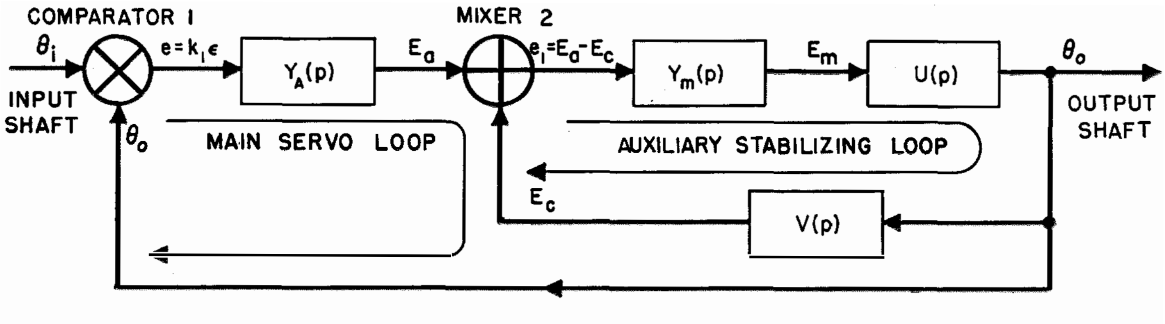


Figure 2—Positioning-type servo mechanism.

k_m = velocity constant in radians per second per volt

$$=(\Delta\Omega_m/\Delta E_m)_{M_m} \tag{20}$$

f_m = motor internal damping characteristic in foot-pound-seconds per radian

$$=-\frac{k_t}{k_m}=\left(-\frac{\Delta M_m}{\Delta\Omega_m}\right)_{E_m} \tag{21}$$

r_m = motor torque-inertia constant in $1/\text{seconds}^2 = M_m/J_m$ (22)

J_l = load inertia in slug-feet^2 (23)

f_l = load viscous-friction coefficient in foot-pound-seconds per radian (24)

F_l = load coulomb friction in foot-pounds (25)

S_l = load elastance in foot-pounds per radian (26)

N = motor-to-load gear ratio $=\theta_m/\theta_l$ (27)

f_o = over-all viscous-friction coefficient referred to the load shaft

$$=f_l+N^2f_m \tag{28}$$

J_o = over-all inertia referred to load shaft

$$=J_l+N^2J_m \tag{29}$$

T_o = over-all time constant in seconds

$$=J_o/f_o. \tag{30}$$

The ideal motor characteristics of Figure 4 are quite representative of direct-current shunt motors. For alternating-current two-phase motors, one phase of which is excited from a constant-voltage source, the curves are valid up to about 40 percent of synchronous speed.

The motor and load-transfer characteristics are given by

$$\theta_o(p)=\frac{(k_t/N)E_m(p)-F_l(p)}{p^2J_o+pf_o+S}. \tag{31}$$

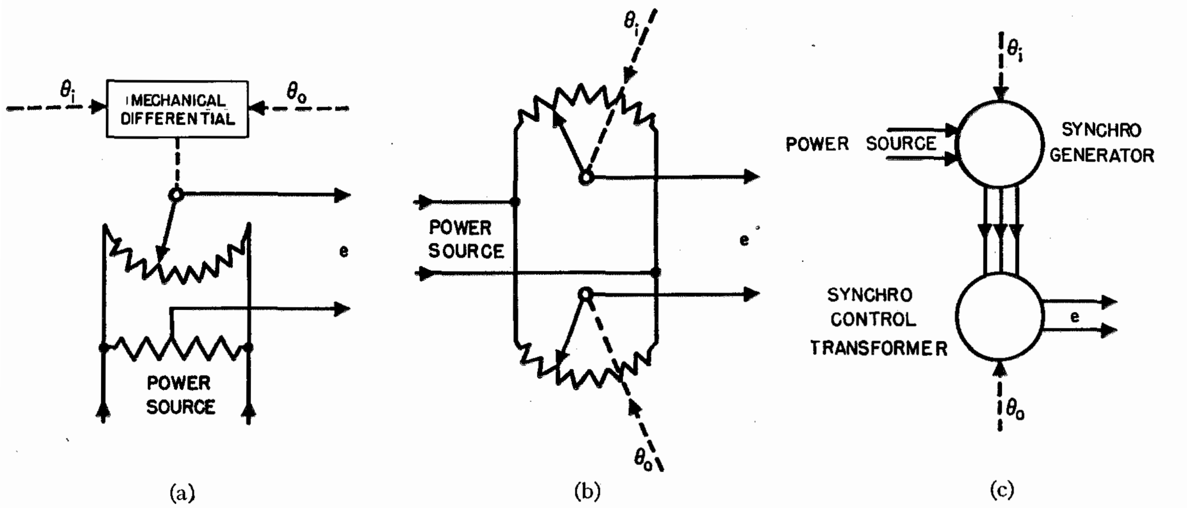


Figure 3—Error-measuring systems.

When $S=0$, which is very often the case,

$$\theta_o(p)=\frac{(k_t/N)E_m(p)-F_l(p)}{p(f_o+pJ_o)}. \qquad (32)$$

and

$$U(p)=\frac{\theta_o(p)}{E_m(p)}$$
$$=\frac{k_t}{N(f_o+pJ_o)p}-\frac{F_l(p)}{E_m(p)(f_o+pJ_o)p}. \qquad (33)$$

When F_l can be assumed zero, then

$$U(p)=\frac{k_t}{N(f_o+pJ_o)p}=\frac{k_t}{Nf_op(T_op+1)}. \qquad (34)$$

4.2 $Y_m(p)$

$Y_m(p)$ represents the power amplifier that energizes the motor system $U(p)$. This amplifier may be of the hard-tube, thyratron, fixed-magnetic, or rotary-magnetic (amplidyne) type. Typical values of $Y_m(p)$ are

$$Y_m(p)=\frac{K_a}{1+pT_a} \qquad (35)$$

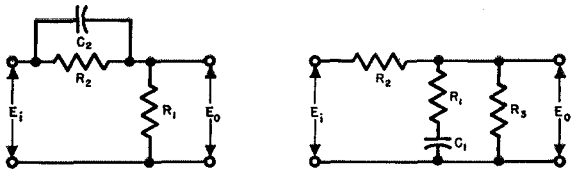
for electronic amplifiers, where T_a is often of negligible magnitude, and

$$Y_m(p)=\frac{K_a}{(1+pT_a)(1+pT_b)} \qquad (36)$$

for a 2-stage magnetic amplifier.

4.3 $Y_A(p)$

$Y_A(p)$ represents the error-voltage amplifier. This amplifier may include various equalizing networks that modify e as required to improve the servo response. Servomechanisms are often classified as shown in Table 1 in accordance with the characteristics of $Y_A(p)$. Practical circuits that approximate some of these characteristics are shown in Figure 5.



proportional+derivative

$$\frac{E_o}{E_i}=\frac{G_o(pT_a+1)}{G_opT_a+1}$$

$$G_o=\frac{R_1}{R_1+R_2}$$

$$T_a=R_2C_2$$

proportional+integral

$$\frac{E_o}{E_i}=\frac{G_o(pT_a+1)}{T_bp+1}$$

$$G_o=\frac{R_3}{R_2+R_3}$$

$$T_a=R_1C_1$$

$$T_b=[R_1+R_3(1-G_o)]C_1$$

Figure 5—Direct-current equalizing networks.

TABLE 1

CLASSIFICATION OF SERVOMECHANISMS BASED ON $Y_A(p)$

$Y_A(p)$	Type of Servomechanism
k_A	Proportional
$k_A(1+pT_a)$	Proportional plus derivative
$k_A\left(1+\frac{1}{pT_a}\right)$	Proportional plus integral
$k_A\left(1+pT_a+\frac{1}{pT_b}\right)$	Proportional plus derivative plus integral

The above circuits are for use where the steady-state error voltage e_{ss} has a direct-current value. In those cases where e_{ss} is a sinusoid of frequency ω_0, the bridged-T circuit is useful as a proportional-plus-derivative network (Figures 6 and 7). For the circuit to possess approximately proportional-plus-derivative characteristics, it is necessary that

$$Y(j\omega)=G[1+jT_d(\omega-\omega_0)]. \qquad (37)$$

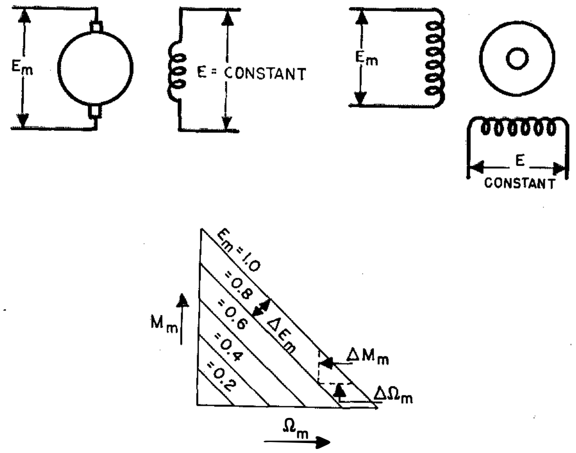


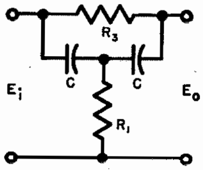
Figure 4—Ideal motor curves.

This is true when

$$R_1=\frac{1}{T_d\omega_0{}^2C},\quad R_3=\frac{T_d}{C},\quad \text{and}\quad G=\frac{2}{T_d{}^2\omega_0{}^2+2}. \tag{38}$$

4.4 $V(p)$

$V(p)$ is a feedback and amplifier network that is used effectively to modify the characteristics



$$\frac{E_o}{E_i}=\frac{T_1T_3p^2+2T_1p+1}{T_1T_3p^2+(2T_1+T_3)p+1}$$
$$T_1=R_1C$$
$$T_3=R_3C$$

Figure 6—Alternating-current derivative network.

of the power amplifier and motor elements. Often this takes the form of a tachometer generator coupled to the output shaft, or equivalent, that develops a voltage e_g proportional to the output-shaft speed. This voltage may be further modified by circuits that are usually of the derivative type. Typical circuits are shown in Figure 8.

5. *Typical Positioning Servomechanisms*

5.1 Simple Viscous-Damped System

For the simple viscous-damped servomechanism, referring to Figure 2,

$$Y_A(p)=k_A,\quad Y_m(p)=1,\quad V(p)=0,$$

and

$$U(p)=\frac{k_t/N}{f_op(T_op+1)}. \tag{39}$$

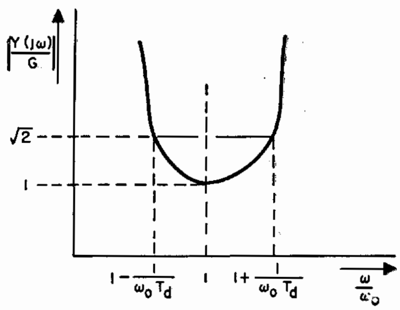


Figure 7—Alternating-current derivative network characteristics.

From (11), we have

$$Y(p)=\frac{k_1k_Ak_t/N}{f_op(T_op+1)}=\frac{|K}{p(T_op+1)}, \tag{40}$$

where

$$|K=\frac{k_1k_Ak_t}{f_oN}\ \text{seconds}^{-1}$$

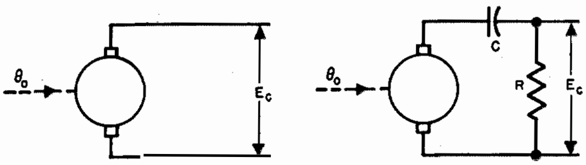
or

$$Y(p)=\frac{|K_m}{J_op(p+1/T_o)}, \tag{41}$$

where $|K_m=|Kf_o$ foot-pounds per radian. Also, from (13),

$$Y_i(p)=\frac{\frac{J_o}{|K_m}\left(p+\frac{1}{T_o}\right)}{1+\frac{J_o}{|K_m}p\left(p+\frac{1}{T_o}\right)}=\frac{p(p+2r\omega_n)}{p^2+2r\omega_np+\omega_n{}^2} \tag{42}$$

$$=\frac{p(p+2r\omega_n)}{\{p+\omega_n[r+(r^2-1)^{\frac{1}{2}}]\}\{p+\omega_n[r-(r^2-1)^{\frac{1}{2}}]\}},$$



(a) (b)

For A, $\frac{E_c(p)}{\theta_o(p)}=pK_g$ and for B, $\frac{E_c(p)}{\theta_o(p)}=K_g\frac{T_gp^2}{pT_g+1}$, $T_g=RC$.

Figure 8—Tachometer feedback network.

where

$$\omega_n=(|K_m/J_o)^{\frac{1}{2}}$$
$$=\text{system natural angular velocity}, \tag{43}$$

$$r=1/(2T_o\omega_n)$$
$$=\text{ratio of actual to critical damping}. \tag{44}$$

For $\theta_i(p)=\omega_i/p^2$ (step-velocity function of amplitude ω_i),

$$\frac{\epsilon(t)}{\theta_{ssc}}=r\left\{1-\epsilon^{-r\omega_nt}\left[\cos\,(1-r^2)^{\frac{1}{2}}\omega_nt+\frac{2r^2-1}{2r(1-r^2)^{\frac{1}{2}}}\sin\,(1-r^2)^{\frac{1}{2}}\omega_nt\right]\right\}, \tag{45}$$

where

$$\theta_{ssc}=2\omega_i/\omega_n$$
$$=\text{steady-state error for critical damping}. \tag{46}$$

Equation (45) is plotted in Figure 9.

5.2 Proportional-Plus-Derivative System

The transfer functions of the proportional-plus-derivative system are identical with those of the proportional system, except that

$$Y_A(p)=k_A(1+pT_A), \quad (47)$$

so that

$$Y(p)=\frac{|K_m}{J_o}\frac{1+pT_A}{p(p+1/T_o)} \text{ and} \quad (48)$$

$$\left.\begin{aligned} Y_i(p)&=\frac{p(p+1/T_o)}{p^2+p\left(\frac{1}{T_o}+\frac{|K_m}{J_o}T_A\right)+\frac{|K_m}{J_o}} \\ &=\frac{p(p+2\omega_n cr)}{p^2+2r\omega_n p+\omega_n^2},\end{aligned}\right\} \quad (49)$$

where

$$\omega_n=(|K_m/J_o)^{\frac{1}{2}} \quad (50)$$

$$\left.\begin{aligned} c&=\frac{1/T_o}{\frac{1}{T_o}+\omega_n^2T_A} \\ &=\text{ratio of viscous to over-all damping and}\end{aligned}\right\} \quad (51)$$

$$r=\frac{1}{2\omega_n}\left(\frac{1}{T_o}+\omega_n^2T_A\right)=\frac{1}{2\omega_n cT_o}. \quad (52)$$

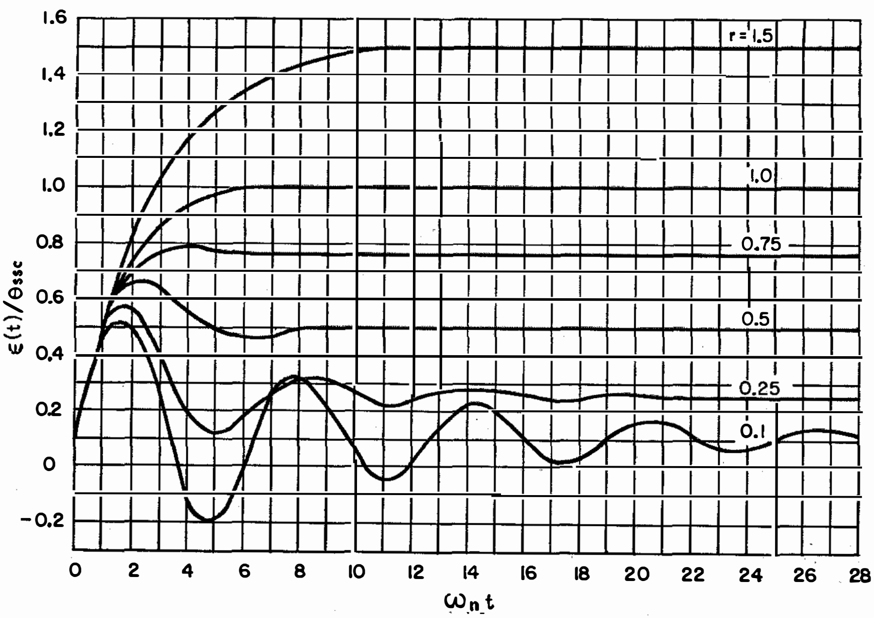


Figure 9—Proportional viscous-damped system.

For $\theta_{i(p)}=\omega_i/p^2$,

$$\epsilon(t)=\frac{2rc\omega_i}{\omega_n}\left\{1-\epsilon^{-r\omega_n t}\left[\cos(1-r^2)^{\frac{1}{2}}\omega_n t+\frac{2r^2c-1}{2rc(1-r^2)^{\frac{1}{2}}}\sin(1-r^2)^{\frac{1}{2}}\omega_n t\right]\right\}. \quad (53)$$

Equation (53) for $c=0$ (i.e., $1/T_o=0$ and $f_o=0$) is plotted in Figure 10.

5.3 Examples of Simple System with Auxiliary Feedback Loop

For a simple system with feedback loop, as shown in Figure 2,

$Y_A(p)=k_A$ and $Y_m(p)=1$.

$$U(p)=\frac{k_t/N}{f_op(T_op+1)}=\frac{k_t/N}{p^2J_o+f_op}$$

$V(p)=k_gp$ for the circuit of Figure 8A.

$=k_gT_gp^2$ for the circuit of Figure 8B, assuming $1\gg pT_g$, so that

$$Y(p)=\frac{\frac{k_Ak_t/N}{p^2J_o+pf_o}}{1+\frac{k_tV(p)}{N(p^2J_o+pf_o)}}=\frac{k_Ak_t/N}{p^2J_o+f_op+\frac{k_t}{N}V(p)}. \quad (54)$$

It is seen therefore that, if $V(p)=k_gp$, the effect is to increase the motor damping to $f_o+k_tk_g/N$.

Similarly, when $V(p)=k_gT_gp^2$, the over-all inertia is effectively increased to $J_o+k_tk_gT_g/N$.

Since k_g can be negative or positive, it follows that $V(p)$ provides a method of effectively decreasing or increasing the damping and inertia.

6. Servomechanism Performance Criteria

It is very difficult to describe completely or specify the performance of servomechanisms. However, the following steady-state quantities and their typical magnitudes may be used as a guide.

Static error ϵ_s

$$=\text{error when input shaft is at rest.} \quad (55)$$

Velocity figure of merit K_V

$$=\omega_i/\epsilon_{ss}=\text{input velocity/error.} \quad (56)$$

Acceleration figure of merit $K\alpha$

$$=\alpha_i/\epsilon_{ss}=\text{input acceleration/error.} \quad (57)$$

Typical performance values are given in Table 2.

TABLE 2

TYPICAL PERFORMANCE VALUES

Quantity	Excellent	Good	Poor
ϵ_s	15 minutes	1 degree	5 degrees
K_V	200 seconds^{-1}	100 seconds^{-1}	25 seconds^{-1}
K_a	150 seconds^{-2}	75 seconds^{-2}	15 seconds^{-2}

7. *Stability Criteria*

A system is unstable when its amplitude of oscillation theoretically increases without limit. Instability is mathematically determined by taking the denominator of $Y_o(p)$ or $Y_i(p)$, in (8) and (9),

$$D=\sum_{i=0}^{i=n} a_i p^i \quad (58)$$

and putting it in the form

$$D=(p+p_0)(p+p_1)\times(p+p_2)\cdots(p+p_n). \quad (59)$$

If any root p_i has a negative real part, the system is then unstable.

The labor involved in transforming (58) into (59) is considerable, particularly when n exceeds 2. To avoid this labor Routh has specified requirements for the coefficients a_i. If these requirements are satisfied, no p_i has a negative real part. These requirements, known as the "Routh stability criteria," are as follows:

A. All coefficients a_i must be positive.
B. A certain relation, depending upon the degree of D, must exist between the coefficients a_i.

For the lower-degree equations, the relations in B above are as follows.

A. For the first and quadratic degrees, the coefficient of p must exceed zero.

B. Cubic, $a_3p^3+a_2p^2+a_1p+a_0$.
For stability, $a_2a_1>a_3a_0$.

C. Quartic, $a_4p^4+a_3p^3+a_2p^2+a_1p+a_0$.
For stability, $a_3a_2a_1>a_3^2a_0+a_1^2a_4$.

D. Quintic, $a_5p^5+a_4p^4+a_3p^3+a_2p^2+a_1p+a_0$.
For stability, $a_2(a_4a_1-a_5a_0)(a_4a_3-a_5a_2)>a_4(a_4a_1-a_5a_0)^2+a_0(a_4a_3-a_5a_2)^2$.

A second method for determining stability is known as the "Nyquist stability criterion." This method consists of obtaining the locus of the loop-transfer function $Y(p)$, (6) in the Y plane for values of $p=j\omega$, where ω varies from $+\infty$ to $-\infty$. If the locus, described in a positive sense, encloses the point -1, 0, the system is unstable. (By positive sense is meant that the interior of the locus is always on the left as the point describes the locus.) Since the locus is always symmetrical about the real axis, it is necessary to draw only the locus for positive values of ω;

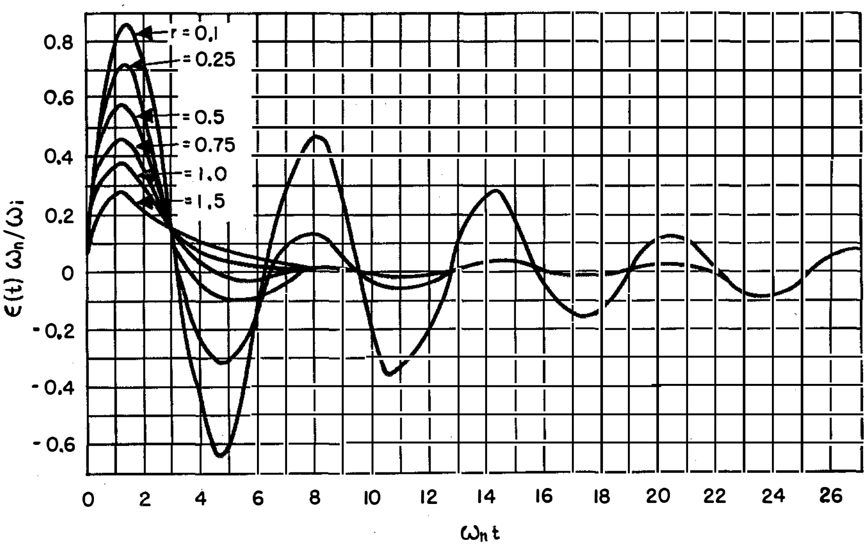
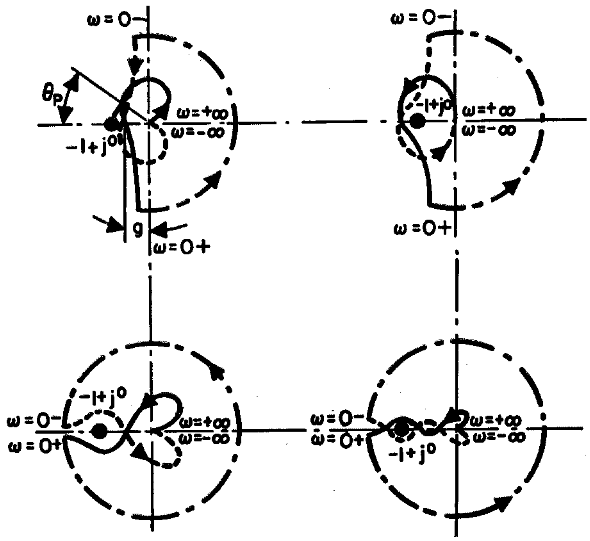


Figure 10—Proportional-plus-derivative system.

the remainder of the locus is then obtained by reflection in the real axis.

Figure 11 shows loci for several simple systems. Curves A and C represent stable systems, curve

(a) (b)



(c) (d)

Solid line = locus for $0 \leqq \omega \leqq \infty$.
Dotted line = locus for $-\infty \leqq \omega \leqq 0$.
Dash-dot line = locus for $\omega = 0$.

Figure 11—Typical Nyquist loci. Plotted in $Y(j\omega)$ plane.

B an unstable system. Curve D is a conditionally stable one; that is, for a particular range of values of $|K$ it is unstable, but it is stable for both larger and smaller values. It is unstable as shown.

Curve A illustrates a zero-displacement-error system; curve C a zero-velocity-error system.

Curve A also demonstrates the phase margin θ_p, and gain margin g. The phase margin is the angle between the negative real axis and the Y vector when $|Y| = 1$. The gain margin is the value of $|Y|$ when the phase angle is 180 degrees. The gain margin is often specified in decibels, so that $g = 20 \log |Y|$. Typical satisfactory values are 15 decibels for g and 50 degrees for θ_p.

8. *Linearity Considerations*

The preceding material applies strictly to linear systems. Actually all systems are nonlinear to some extent. This nonlinearity may cause serious deterioration in performance. Common sources of nonlinearity are:

A. Nonlinear motor characteristics.

B. Overloading of amplifiers by noise.

C. Static friction.

D. Backlash in gears, potentiometers, etc. For good performance, it is recommended that the total backlash should not exceed 20 percent of the expected static error.

E. Low-efficiency gear or worm drives that cause locking action.

Despite all the available types and sources of nonlinearity, it is usually found that, when care is taken to minimize it, the linear theory applies quite well.

Signal-to-Noise-Ratio Improvement in a Pulse-Count-Modulation System*

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IT IS SHOWN for a pulse-count-modulation system that the output signal-to-noise power ratio expressed in decibels is approximately equal to twice the input signal-to-noise power ratio. It is independent of the number of code pulses when a sufficient number are used. The distortion due to quantization varies greatly with the number of levels. Adopting the criterion that the output noise power should equal the distortion power, a relation giving corresponding values of the number of digits and input signal-to-noise ratio is found.

• • •

1. Introduction

One of the newer methods of transmitting information by means of pulses is known as pulse-count or pulse-code modulation. Several[1–3] papers have discussed the advantages of pulse-count-modulation from the point of view of distortion and cross talk. The purpose of this paper is to discuss another significant aspect of any communication system, which is the system signal-to-noise improvement.

2. General Consideration of Noise

Noise may be divided into two general types; namely, fluctuation noise and impulse noise. In what follows, however, only fluctuation noise will be considered. Fluctuation noise is assumed to be composed of an infinite number of equal infinitesimal components covering the whole frequency spectrum, and assumed to have a random and continually varying phase.

If fluctuation noise is observed for a sufficiently long time, it will be found that any given voltage will be exceeded for a certain fraction of the time of observation. The differential probability that the instantaneous noise voltage lies between V and $V+dV$ is

$$dp=\frac{1}{(2\pi)^{1/2}e}\exp(-V^2/2e^2)dV$$
$$=\frac{1}{(2\pi)^{1/2}e}\exp(-r^2/2)dV, \tag{1}$$

where e is the root-mean-square value of noise voltage and $r=V/e$.

This is often expressed by saying that the amplitude of the noise voltage is distributed normally about the mean value.

If the circuit includes an ideal low-pass filter, with a cutoff frequency f_0, it has been shown by Rice[4] that the probable number of positive noise bursts per second of amplitude greater than V is

$$n(V)=\frac{f_0}{3^{1/2}}\exp(-r^2/2). \tag{2}$$

There is an equal probable number of negative bursts exceeding V in amplitude.

An approximate derivation of (2) is given in the Appendix.

The noise bursts will be considered in what follows as being independent of each other. Further, the number of noise bursts in a given interval of time will be considered as independent from what occurs in any other interval. Of course, over a sufficiently great period of time, the average number $n(V)$ of noise bursts per second above V is determined. Accordingly, we may assume that the distribution of noise bursts

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[1] A. G. Clavier, P. F. Panter, and D. D. Grieg, "PCM Distortion Analysis," *Transactions of the AIEE* (*Electrical Engineering*, November, 1947), v. 66, pp. 1110–1122; November, 1947.

[2] H. S. Black and J. O. Edson, "PCM Equipment," *Transactions of the AIEE* (*Electrical Engineering*, November, 1947) v. 66, pp. 1123–1124; November, 1947.

[3] D. D. Grieg, "Pulse Count Modulation System," *Tele-Tech*, v. 6, pp. 48–52; September, 1947.

[4] S. O. Rice, "Mathematical Analysis of Random Noise," *Bell System Technical Journal*, v. 23 pp. 282–332; July, 1944.

obeys Poisson's law, which states that the probability of obtaining exactly k disturbances in an interval of time T is

$$p(k) = \frac{(nT)^k}{k!} \exp(-nT). \tag{3}$$

As shown in Figure 1, the curve of $p(k)$ versus k assumes a variety of forms according to the value of nT. If nT is small—say, 0.1—then $p(o)$ is nearly unity and $p(k)$ decreases rapidly as k increases. When nT is large—say, 100—the curve is sharply peaked at $k=100$.

3. Application to Pulse-Count Modulation

To evaluate the effect of noise in the video-frequency section of the receiver, it will be recalled that in a binary pulse-count-modulation system information is conveyed by a given number μ of code pulses, giving in all $2^\mu - 1$ discrete amplitude levels for the output pulse-amplitude-modulated pulses. These pulse-amplitude-modulated pulses are subsequently converted to audio frequencies by means of a low-pass filter. If the amplitude of the code pulses is taken as $2V$, the following simplifying assumptions will be made as to the effect of noise bursts:

1. A negative noise burst of any shape or duration, and of amplitude greater than V, when combined with a code pulse, will cause complete obliteration of the code pulse.

2. A positive noise burst of any shape or duration, and of amplitude greater than V, occurring in the absence of a code pulse, will cause a spurious code pulse to appear.

These assumptions are justified by (26) of the Appendix, which shows that the average duration of a noise burst above V is small compared with the width of the pulse-amplitude-modulated pulse, provided the cutoff frequency f_0 of the filter is chosen as low as the duration τ of the code pulse will allow (that is to say, $\frac{1}{2}\tau$), and the input signal-to-noise ratio is sufficiently large. Hence, a noise burst will cause the insertion or cancellation of only one code pulse, except for very rare cases which may be neglected. Of course, when noise power is comparable to the signal power this is no longer true, but the system would then become inoperative.

For the time being, suppose that only one noise burst occurs during the time T allotted for the transmission of the μ code pulses. In what follows, the interval T will be referred to as the subframe. As a noise burst may occur in any of the μ positions, the mean square change in the level of the output pulse-amplitude-modulated pulses is

$$M_1 = \frac{1}{\mu} \sum_1^\mu 2^{2(\mu-1)} = \frac{4^\mu - 1}{3\mu}. \tag{4}$$

This is expressed in terms of the square of the value of one level taken as an arbitrary unit, and consequently has the dimension of a voltage squared.

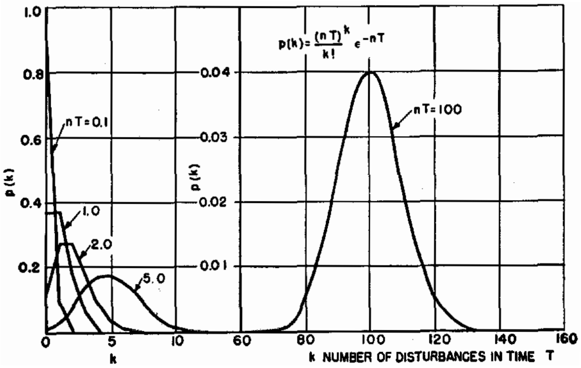

Figure 1—Curves of Poisson's law.

Taking account of the fact that on the average there is only half of the noise bursts (positive or negative) which effectively alter the code, the other half being composed of those positive bursts that fall simultaneously with a code pulse, and those negative noise bursts that occur when there is no pulse, the probability of obtaining one effective noise burst in a subframe is $p(1)$, by (3), and the output noise power (for a unit resistor) of the output pulse-amplitude-modulated pulses is, therefore,

$$N_1 = \frac{4^\mu - 1}{3\mu} \frac{\tau}{T} p(1), \tag{5}$$

where τ is the duration of an output pulse-amplitude-modulated pulse, and (1) indicates that there is only one noise burst per subframe.

Provided the input signal-to-noise ratio is sufficiently large, the quantity nT in Poisson's equation (3) will be small compared to unity, and, as previously mentioned, the probability $p(k)$ decreases rapidly as k increases. The case of two noise bursts in a subframe is thus already very much less probable than the case of only one burst, for all practical cases of pulse-count-modulation transmission systems. An approximate calculation of the contribution of this case to the output noise is obtained as follows. Let two indices (i, j) designate the position of the modified code pulses. The amplitude of the output pulse is modified by the burst (i, j) in any of the following ways:

$$2^i+2^j,\ 2^i-2^j,\ -2^i+2^j,\ -2^i-2^j,$$

depending on the particular subframe being considered. Assuming that any of these combinations is equally probable, the mean square variation of the corresponding output pulse-amplitude-modulated pulses is

$$\begin{aligned} m_{i,j} &= \tfrac{1}{4}[(2^i+2^j)^2+(2^i-2^j)^2 \\ &\quad +(-2^i+2^j)^2+(-2^i-2^j)^2] \\ &= 4^i+4^j. \end{aligned}$$

It is now necessary to sum $m_{i,j}$ over all possible values of (i, j), noting, however, that the burst (i, i) is not more damaging than the single burst

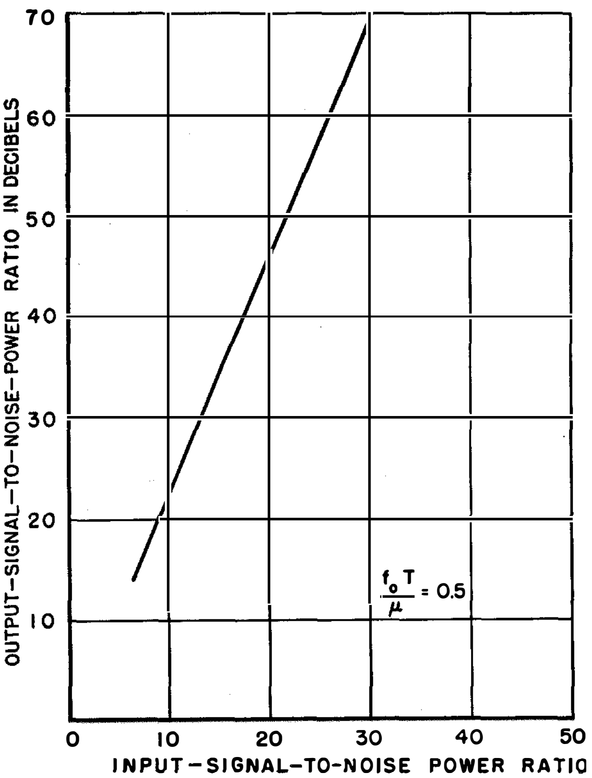

Figure 2—Output-signal-to-noise ratio versus input signal-to-noise ratio.

i. The result is

$$M_2=\sum_{i=1}^{i=\mu}\sum_{j=1}^{j=\mu} m_{i,j}=\frac{2\mu-1}{\mu}M_1. \tag{6}$$

If μ is large, this is very close to

$$M_2=2M_1. \tag{7}$$

A similar reasoning may be applied to three or more bursts per frame; in fact, so long as the number of bursts per frame does not become equal to the number of code pulses. As the probability for this to occur is negligible if the signal-to-noise ratio is large enough for the system to be operative, the output noise power (for a unit resistor) is closely approximated by the following expression:

$$\begin{aligned} N_0 &= \frac{4^\mu-1}{3\mu}\frac{\tau}{T}[p(1)+2p(2)+3p(3)+\cdots] \quad (8) \\ &= \frac{4^\mu-1}{3}\frac{n(V)\tau}{\mu}. \end{aligned}$$

The number of noise bursts $n(V)$ is given by (2), so that

$$N_0=\frac{4^\mu-1}{3\cdot 3^{1/2}}\frac{\tau}{T}\left(\frac{f_0T}{\mu}\right)\exp(-r^2/2). \tag{9}$$

The output signal power is determined by the amplitude and form factor of the quantized signal. For a sinusoid of peak value $m/2(2^\mu-1)$ where m is the degree of modulation, this output signal power (in a unit resistor) is[1]

$$S_0=\frac{1}{8}m^2\frac{\tau}{T}(2^\mu-1)^2. \tag{10}$$

The output signal-to-noise ratio is

$$(S/N)_0=\frac{3\cdot 3^{1/2}m^2}{8}\cdot\frac{2^\mu-1}{2^\mu+1}\left(\frac{\mu}{f_0T}\right)\exp(r^2/2). \tag{11}$$

Noting that T/μ is the duration of a code pulse, we see that f_0T/μ is a significant parameter of a pulse system that determines the shape of the pulse. An acceptable minimum value for this quantity is 0.5. We may write further that

$$r^2=\frac{V^2}{e^2}=(S/N)_I, \tag{12}$$

where $(S/N)_I$ is the input signal-to-noise ratio. Thus the final result is

$$(S/N)_0 \simeq 1.3m^2\exp[\tfrac{1}{2}(S/N)_I], \tag{13}$$

where both $(S/N)_I$ and $(S/N)_0$ are expressed as power ratios. Letting $m=1$ and expressing $(S/N)_0$ in decibels gives the relation

$$(S/N)_{0\text{db}} \simeq 2.2 (S/N)_{I_{\text{power}}}. \tag{14}$$

Thus the output signal-to-noise power ratio expressed in decibels is proportional to the input signal-to-noise ratio expressed as a power ratio. This result is independent of the number of code pulses, provided enough are used; that is, more than 3 or 4. The results of (14) are shown graphically in Figures 2 and 3.

In any pulse-count-modulation transmission system, although the effect of noise is substantially independent of the number of code digits, the percent distortion is, of course, directly influenced by it. The distortion power of the output pulses is given by[1]

$$D = \frac{A^2}{12N^2}\frac{\tau}{T} = \frac{1}{12}\frac{\tau}{T}, \tag{15}$$

where

$2A$ = peak-to-peak value of signal $= 2^\mu - 1$
$2N$ = total number of levels $= 2^\mu - 1$.

Taking $f_0 T/\mu = 0.5$, the output noise power is, from (9),

$$N_0 = \frac{4^\mu - 1}{6 \cdot 3^{1/2}} \frac{\tau}{T} \exp\left[-\tfrac{1}{2}(S/N)_I\right]. \tag{16}$$

Thus,

$$\frac{D}{N_0} = \frac{3^{1/2}}{2}\frac{1}{4^\mu - 1}\exp\left[\tfrac{1}{2}(S/N)_I\right] \simeq \frac{1}{4^\mu}\exp\left[\tfrac{1}{2}(S/N)_I\right]. \tag{17}$$

The output noise power will thus be equal to the distortion power when

$$\exp\left[\tfrac{1}{2}(S/N)_I\right] = 4^\mu.$$

A curve of $(S/N)_0$ in decibels versus μ is plotted in Figure 4 under these conditions. It gives corresponding values of output signal-to-noise ratio and number of code pulses for equality of the distortion and output noise power.

This criterion is useful in the design of cable systems where the signal power is constant. It gives the highest fidelity consistent with other factors. For instance, about 11 digits are required for an output signal-to-noise and distortion ratio of 60 decibels. The improvement secured by a further increase in the number of digits would be small and not warranted by the added complexity.

4. *Effect of Noise on Pulse-Count-Modulation Systems with Repeating Stations*

Let the relay system in Figure 5 consist of k identical linear amplifiers and k identical paths, the gain of an amplifier being just sufficient to overcome the path attenuation. The various noise sources of root-mean-square value e may then be replaced by an equivalent noise source of root-mean-square value $k^{1/2}e$ at the input. Equation (14) gives

$$(S/N)_{0\text{db}} = \frac{2.2}{k}(S/N)_I, \tag{18}$$

where $(S/N)_I$ is the input signal-to-noise power ratio as defined previously; the output signal-to-

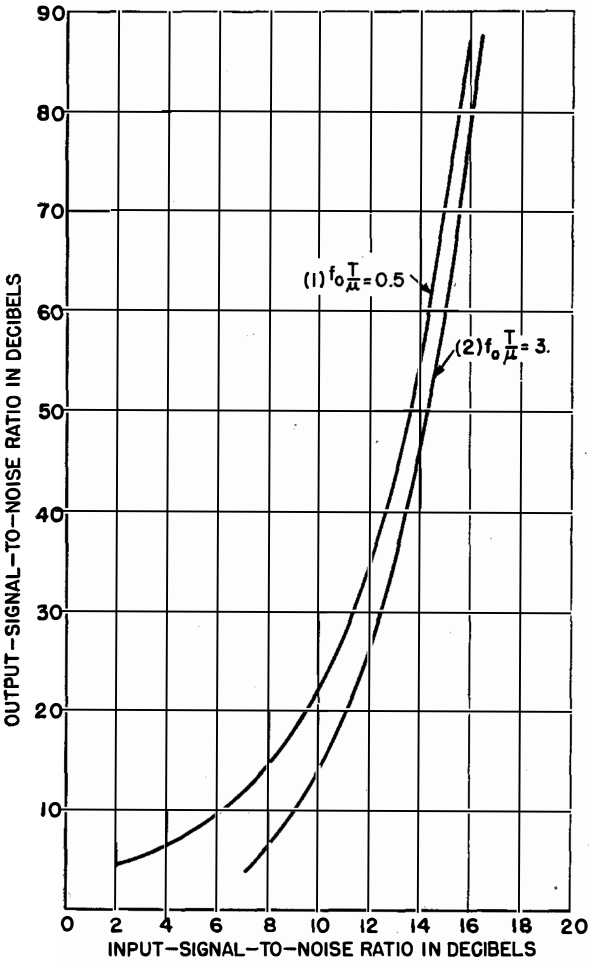

Figure 3—Signal-to-noise improvement in a pulse-count-modulation system (S/N) output decibels versus (S/N) input decibels

noise ratio again is expressed in decibels. In practice, the results obtained may be somewhat worse due to the progressive deterioration of the pulse shape.

In another type of relay system, the repeaters regenerate the code pulses so that the output consists of idealized pulse-count-modulated pulses. The gain, again, is just sufficient to overcome the path attenuation. As noise enters

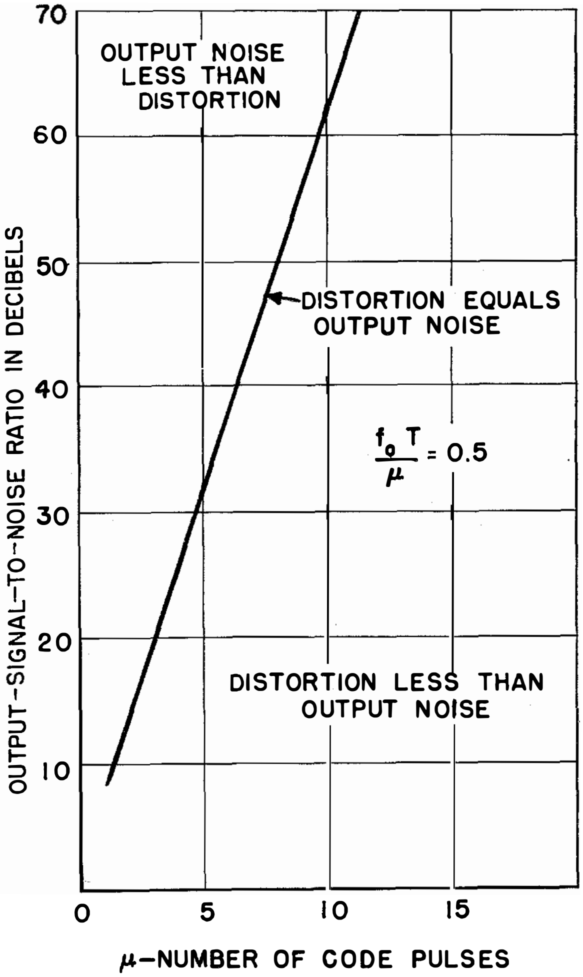

Figure 4 — Output-signal-to-noise ratio versus number of code pulses when distortion equals noise.

in the form of wrong code pulses, each noise source contributes n noise bursts, and consequently n wrong codes independently of the others. If we neglect the possibility of some noise bursts canceling the effect of others, (9) for the output noise power becomes

$$N_0 = k\frac{4^\mu - 1}{3\cdot 3^{1/2}}\frac{\tau}{T}\left(\frac{f_0 T}{\mu}\right)\exp(-r^2/2).$$

Using this value, (14) becomes

$$(S/N)_{0\text{db}} = 2.2(S/N)_I - 10\log k. \qquad (19)$$

From this we see that the curve of Figure 2 is moved parallel to itself, whereas in the first case considered its slope is changed.

To show the effect more clearly, the input signal-to-noise ratio for an assumed output signal-to-noise ratio of 60 decibels versus the number of repeaters is plotted in Figure 6. It is seen that, for the linear repeaters, the input level rises considerably with the number of repeaters, and requires a 17-decibel increase for 50 repeaters. On the other hand, for regenerative repeaters a small increase of 1 decibel takes care of the cumulative effect of noise. This represents a considerable economy in total power installed, since each repeater power must necessarily be increased. Systems combining the two types may be used, thereby obtaining intermediate results.

5. *Conclusions*

A study of the effect of fluctuating noise on pulse-count-modulation signals shows that the output signal-to-noise power ratio expressed in decibels is proportional to the input signal-to-noise power ratio. It is substantially independent of the number of code digits when a sufficient number is used, superior to, say, 3. The distortion due to quantization, however, varies greatly with the number of levels, and consequently the number of code digits. A relation is given showing the number of digits for which the output

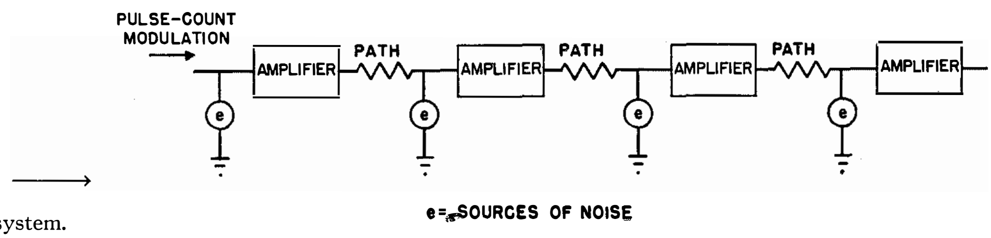

Figure 5—Relay system.

noise power is equal to the distortion power, when a definite input or output signal-to-noise ratio is assumed. It is, of course, unnecessary to increase the number of digits beyond that value for which these two powers are equal.

In case the communication link includes a number of relays, pulse-count modulation shows a striking advantage over other modulation systems when use is made of regenerative repeaters. A very small increase in input signal-to-noise ratio accounts for the cumulative effect of noise along the whole chain of repeaters.

6. *Acknowledgment*

Acknowledgment is made to G. Deschamps for his mathematical assistance in the preparation of this paper.

7. *Appendix*

An approximate derivation of (2) will be given here. If wide-band fluctuation noise is passed into an ideal low-pass filter, the large peaks on the input side may be considered as being essentially isolated impulses. Thus, the output noise peaks are determined by the impulse response of the filter, which is

$$v(t) = V_p \frac{\sin 2\pi f_0 t}{2\pi f_0 t}, \tag{20}$$

where V_p is the peak voltage and f_0 the cutoff frequency. From the definition of probability $p(V)$, we have

$$p(V) = n(V) \cdot t(V), \tag{21}$$

where $n(V)$ is the number of peaks above V, and $t(V)$ is the average duration of such peaks.

On the other hand, the expression of $p(V)$ is

$$p(V) = \frac{1}{(2\pi)^{1/2} e} \int_V^\infty \exp(-V^2/2e^2) dV.$$

Utilizing an asymptotic series for the integral, it is thus found by limiting the expansion to two terms that $p(V)$ is expressed by

$$p(V) = \frac{1}{(2\pi)^{1/2}} \frac{e}{V} \left(1 - \frac{e^2}{V^2}\right) \exp(-V^2/2e^2) dV. \tag{22}$$

The average duration $t(V)$ may be found by replacing the noise peaks above V by an average noise peak of height $\bar{V}$ given by

$$\bar{V} = \frac{\frac{1}{(2\pi)^{1/2} e} \int_V^\infty V \exp(-V^2/2e^2) dV}{\frac{1}{(2\pi)^{1/2} e} \int_V^\infty \exp(-V^2/2e^2) dV}. \tag{23}$$

To the approximation of (22), this gives

$$\bar{V} = \frac{V}{1 - \frac{e^2}{V^2}} \simeq V\left(1 + \frac{e^2}{V^2}\right). \tag{24}$$

The average duration $t(V)$ is then found by solving

$$V = V\left(1 + \frac{e^2}{V^2}\right) \frac{\sin \pi f_0 t(V)}{\pi f_0 t(V)}. \tag{25}$$

Assuming $\pi f_0 t(V)$ is a small angle, the result is

$$t(V) = \frac{6^{1/2}}{\pi f_0} \frac{e}{V} \left(1 - \frac{e^2}{2V^2}\right). \tag{26}$$

The average number of noise peaks is then derived from (21), and is found to be

$$n(V) = \frac{\pi^{1/2}}{2} \frac{f_0}{3^{1/2}} \left(1 - \frac{e^2}{2V^2}\right) \exp(-V^2/2e^2). \tag{27}$$

This is very nearly the result given by Rice and utilized in the text.

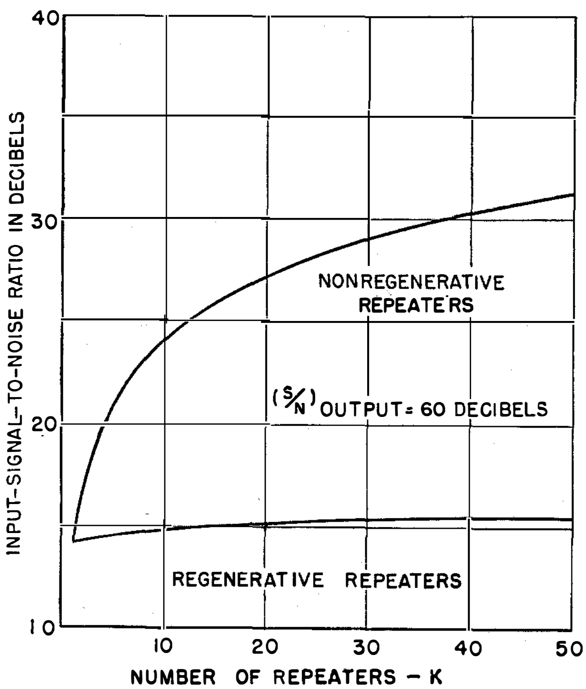

Figure 6—Input-signal-to-noise ratio versus number of repeaters for constant output signal-to-noise ratio.

Contributors to This Issue

J. H. BRUNDAGE

JOHN H. BRUNDAGE was born in Orlando, Florida, on November 9, 1917. He received the B.S.E.E. degree with honors from the University of Florida in 1939.

He served as an assistant engineer with the Orlando Utilities Commission until 1940 and then became a member of the staff of the United States Army Signal Corps Laboratories at Fort Monmouth, New Jersey.

In 1944, Mr. Brundage was employed by Federal Telephone and Radio Corporation. He is now engaged in the development of voice-frequency and carrier repeaters and carrier telephone and telegraph terminal equipment.

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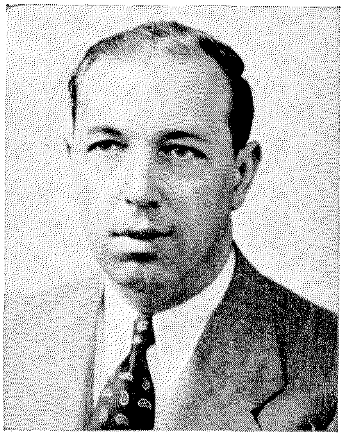
HENRI BUSIGNIES was born in Sceaux, France on December 29, 1905. He received the degree of Electrical Engineer from Paris Univeristy in 1926.

On completion of his military service, he entered the Paris Laboratories of the International Telephone and Telegraph Corporation in 1928. Until about 1940, Mr. Busignies traveled to Italy, Spain, Switzerland, England, Africa, and the United States in the interests of the company. Since 1940, he has been an executive engineer and is now the Director of Federal Telecommunication Laboratories in New York City.

His first patent was issued in 1926 and has been followed by almost a hundred others. They have been dominantly in the field of automatic direction finding with particular attention to aircraft applications. He received the Lakhovsky award of the Radio Club of France in 1926.

Mr. Busignies is a Fellow of the Institute of Radio Engineers.



HENRI BUSIGNIES

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A. G. CLAVIER

A. G. CLAVIER was born in Cambrai, France, in 1894. He received a degree in electrical engineering from Ecole Supérieure d'Electricité in 1919 and then joined the staff of engineers organized by General Ferrié at the Etablissement Central de la Radiotélégraphie Militaire. He was in charge of research on high frequencies from 1920 to 1925.

In 1929, Mr. Clavier joined Les Laboratoires Standards in Paris which later became Laboratoire Central de Télécommunications, and has been continuously engaged in research on centimeter and millimeter waves. He was in charge of the experiments which, in 1930, resulted in 17-centimeter-wave transmission across the English Channel and of the developments for the Lympne-St. Inglevert microwave radiotelephone link, which was inaugurated commercially in 1934. He was assistant director of research in 1945, when he was transferred to Federal Telecommunication Laboratories in New York, where he now holds the same position.

He has published extensively on high-frequency oscillators, wave guides, and general electromagnetic theory, and has taught field theory and applications of ultra-high frequencies at the Ecole Supérieure d'Electricité.

Mr. Clavier is president of the section of the Société des Radioélectriciens dealing with hyperfrequencies. He is a Fellow of the Institute of Radio Engineers, and a Member of the Institution of Electrical Engineers.

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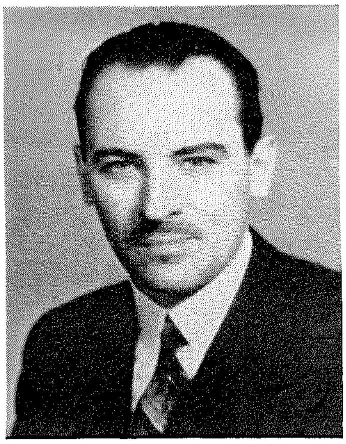

E. M. DELORAINE

E. M. DELORAINE was born in Paris, France, on May 16, 1898. He received the B.S. degree, the Certificat de Mathématiques in 1918, and of Ingénieur diplomé de L'école de Physique et Chimie, a branch of Paris University, in 1920.

In 1917 he joined the French Army Signal Corps, and later engaged in research work at the Eiffel Tower. He became associated with the London engineering staff of the International Western Electric Company in 1921 and began technical work in connection with broadcasting at the experimental station 2WP. Until 1925 he was responsible for part of the developments in Great Britain in connection with the first transatlantic telephone circuit. He was made European technical director of International Standard Electric Corporation in 1933. From 1931 to 1937, Mr. Deloraine made important contributions in the development of ultra-high frequencies. He was also active in the advancement of high-power broadcasting, and established the Prague station with 120 kilowatts carrier, followed by the Budapest station. Mr. Deloraine was also successful in directing experiments in connection with automatic radio compasses for aircraft.

Mr. Deloraine came to the United States in 1941 to take charge of the organization of the laboratories unit for the Federal Telephone and Radio Corporation. In 1945, he was appointed president of International Telecommunication Laboratories, Inc. and in 1946 technical director of the International Telephone and Telegraph Corporation as well as vice president and technical director of the International Standard Electric Corporation.

Mr. Deloraine was made a Chevalier of the Legion of Honor in 1938 for exceptional services to the Posts and Telegraphs Department of France, and he was elected vice-president of the French Institute of Radio Engineers in 1939. He has been a member of the International Consultative Committee of Long Distance Telephony since 1927, and is also a member of the French Astronomical Society. Mr. Deloraine is a Fellow of the Institute of Radio Engineers and was its vice-president for 1946. He is a Fellow of the American Institute of Electrical Engineers.

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MILTON DISHAL

MILTON DISHAL was born on March 20, 1918, in Philadelphia, Pennsylvania. Temple University conferred on him two degrees, the B.S. in 1939 and M.A. in 1941. He was a Teaching Fellow in physics in that University from 1939 to 1941.

Mr. Dishal entered Federal Telecommunication Laboratories in 1941 and is now a senior engineer in the development of radio receivers having special characteristics.

Mr. Dishal is a Senior Member of the Institute of Radio Engineers.

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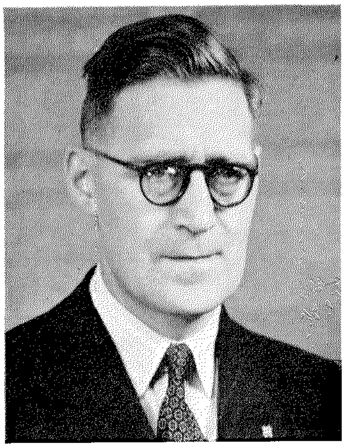

D. H. ORMROD

D. H. ORMROD was born in London, England. He joined Standard Telephones and Cables, Limited, in 1925.

For eight years, he was engaged in the installation of manual-to-automatic telephone exchanges. In 1933, he was assigned to the development of test equipment for factory and installation use. During the war, he handled miscellaneous service developments and is now designing and developing circuits for automatic switching systems.

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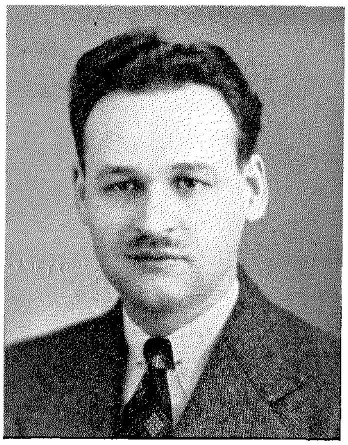
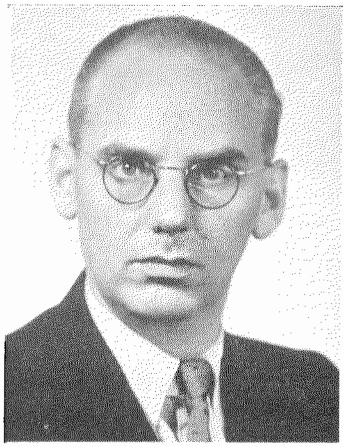
BENJAMIN PARZEN was born on April 5, 1913 in Poland. He received the



BENJAMIN PARZEN



F. W. RABE

B.S. degree in engineering from the College of the City of New York in 1936.

During 1936 and 1937, he was a radio inspector for the Federal Communications Commission. From 1938 to 1944, he served as a civilian engineer for the United States Navy. He joined Federal Telecommunication Laboratories in 1944 and is now a senior engineer.

Mr. Parzen is a member of the American Institute of Electrical Engineers and is licensed as a professional engineer by New York State.

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FREDERIK WILLEM RABE was born on May 8, 1900, in Utrecht, Netherlands. He received an electrical engineering degree from the Institute of Technology (Technische Hoogeschool) at Delft in 1922.

In 1925, he joined the technical staff of Bell Telephone Manufacturing Company in Antwerp to work on automatic switching systems.

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H. STANESBY

H. STANESBY was born in London, England, on August 2, 1906. He received his education at Emanuel School, Wandsworth Technical Institute, and London University.

In 1925, he became an inspector in the engineering department of the British Post Office, later being engaged at the experimental laboratories in the development of the transatlantic telephone. In 1930, he became assistant engineer and worked on radio receivers, crystal filters, and terminal equipment for coaxial cables.

Mr. Stanesby was promoted to executive engineer in 1938 and, from 1944 to 1947, served at Post Office headquarters, representing that organization at the Second International Meeting on Radio Aids to Marine Navigation held in the United States. He is now assistant staff engineer at the radio laboratories, being responsible for the development of systems for transmission of television signals over cables.

He is a member of the Institution of Electrical Engineers.

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W. K. WESTON

WILLIAM K. WESTON received the B.Sc. engineering degree from London University in 1913. He then joined the the Western Electric Company for engineering and installation work on the phantom-loaded telephone cable being laid between London and Birmingham, the first of its type in Europe.

In 1921, he was transferred to the general manufacturing department and was active in the program of expanding the European cable network. He returned to Woolwich in 1931 and is now head of the telephone line division of Standard Telephones and Cables, Limited. He is chairman of the Committee on Cables and Wires of the International Telephone and Telegraph System.

Mr. Weston is a Member of the Institution of Electrical Engineers.

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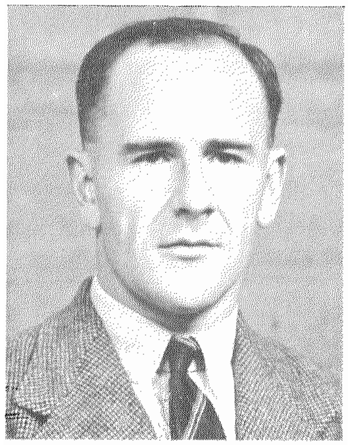
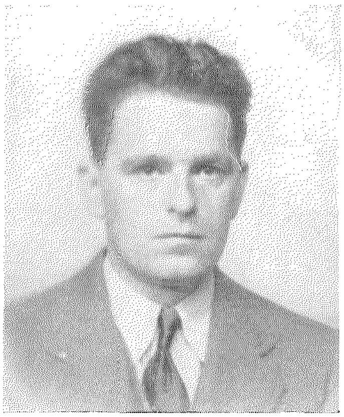


M. T. WILSON

M. T. WILSON was born in Greenock, Scotland, and received his technical education at the Regent Street and Northampton Polytechnic Institutes.

In 1936, he entered the employ of Standard Telephones and Cables, Limited, handling circuit design of telephone exchange equipment and automatic ticketing.



Karl H. Zimmermann

From 1939 to 1946, he served in the Royal Signals, seeing action in the Middle East and Mediterranean theaters. Since then, he has been in the circuit laboratory on the development of subscriber's line identification and automatic ticketing.

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Karl H. Zimmermann was born in Woodside, New York, on January 17, 1911. He received the degree of B.S. in electrical engineering in 1934 from New York University. He holds a professional engineer license granted by the State of New York.

From 1934 to 1937, he served as a construction engineer for the Elite Electric Company of Forest Hills, New York, and for the next five years was with Habirshaw Cable and Wire Corporation in Yonkers, New York, where he was engaged in research, development, and design of communication and power cable.

In 1942, Mr. Zimmermann joined the Intelin Division of Federal Telephone and Radio Corporation. His present duties concern problems relating to the design, manufacture, and sale of power, communication, and high-frequency cables.

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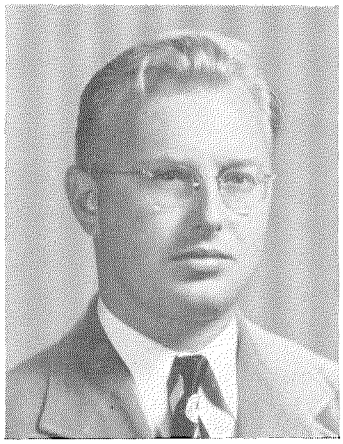

J. Zyda

J. Zyda was born on September 19, 1925, in Vaux Hall, New Jersey. He received the B.S. degree in electrical engineering from Newark College of Engineering in 1945 and an M.S. from Stevens Institute of Technology in 1949.

He joined the wire transmission engineering department of Federal Telephone and Radio Corporation in 1945.

Mr. Zyda is a member of Alpha Sigma Phi and of Tau Beta Pi.

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For a biography and photograph of F. H. Bray, see v. 26, p. 101; March, 1949: for W. Dite and P. F. Panter, see v. 26, p. 181; June, 1949.

INTERNATIONAL TELEPHONE AND TELEGRAPH CORPORATION

Associate Manufacturing and Sales Companies

United States of America

International Standard Electric Corporation, New York, New York

Federal Telephone and Radio Corporation, Clifton, New Jersey

International Standard Trading Corporation, New York, New York

Capehart-Farnsworth Corporation, Fort Wayne, Indiana

Great Britain and Dominions

Standard Telephones and Cables, Limited, London, England

Branch Offices: Birmingham, Leeds, Manchester, England; Glasgow, Scotland; Dublin, Ireland; Cairo, Egypt; Calcutta, India; Johannesburg, South Africa

Creed and Company, Limited, Croydon, England

International Marine Radio Company Limited, Croydon, England

Kolster-Brandes Limited, Sidcup, England

Standard Telephones and Cables Pty. Limited, Sydney, Australia

Branch Offices: Melbourne, Australia; Wellington, New Zealand

Silovac Electrical Products Pty. Limited, Sydney, Australia

Austral Standard Cables Pty. Limited, Melbourne, Australia

New Zealand Electric Totalisators Limited, Wellington, New Zealand

Federal Electric Manufacturing Company, Ltd., Montreal, Canada

South America

Compañía Standard Electric Argentina, Sociedad Anónima, Industrial y Comercial, Buenos Aires, Argentina

Standard Electrica, S.A., Rio de Janeiro, Brazil

Compañía Standard Electric, S.A.C., Santiago, Chile

Europe and Far East

Vereinigte Telephon und Telegraphenfabriks Aktiengesellschaft Czeija, Nissl and Company, Vienna, Austria

Bell Telephone Manufacturing Company, Antwerp, Belgium

Standard Electric Doms a Spolecnost, Prague, Czechoslovakia

China Electric Company, Limited, Shanghai, China

Standard Electric Aktieselskab, Copenhagen, Denmark

Compagnie Générale de Constructions Téléphoniques, Paris, France

Le Matériel Téléphonique, Paris, France

Les Téléimprimeurs, Paris, France

Ferdinand Schuchhardt Berliner Fernsprech und Telegraphenwerk Aktiengesellschaft, Berlin, Germany

C. Lorenz, A.G. and Subsidiaries, Berlin, Germany

Mix & Genest Aktiengesellschaft and Subsidiaries, Berlin, Germany

Süddeutsche Apparatefabrik Gesellschaft m.b.H., Nuremberg, Germany

Telephonfabrik Berliner A.G. and Subsidiaries, Berlin, Germany

Nederlandsche Standard Electric Maatschappij N.V., Hague, Netherlands

Dial Telefonkereskedelmi Részvény Társaság, Budapest, Hungary

Standard Villamossági Részvény Társaság, Budapest, Hungary

Telefongyár r.t., Budapest, Hungary

Fabbrica Apparecchiature per Comunicazioni Elettriche, Milan, Italy

Standard Elettrica Italiana, Milan, Italy

Standard Electric Aktieselskap, Oslo, Norway

Standard Telefon og Kabelfabrik A/S, Oslo, Norway

Standard Electrica, Lisbon, Portugal

Compañía Radio Aérea Marítima Española, Madrid, Spain

Standard Electrica, S.A., Madrid, Spain

Aktiebolaget Standard Radiofabrik, Stockholm, Sweden

Standard Telephone et Radio S.A., Zurich, Switzerland

Telephone Operating Systems

Compañía Telefónica Argentina, Buenos Aires, Argentina

Compañía Telegráfico-Telefónica Comercial, Buenos Aires, Argentina

Compañía Telegráfico-Telefónica del Plata, Buenos Aires, Argentina

Companhia Telefonica Paranaense S.A., Curitiba, Brazil

Companhia Telefonica Rio Grandense, Porto Alegre, Brazil

Compañía de Teléfonos de Chile, Santiago, Chile

Compañía Telefónica de Magallanes S.A., Punta Arenas, Chile

Cuban American Telephone and Telegraph Company, Havana, Cuba

Cuban Telephone Company, Havana, Cuba

Mexican Telephone and Telegraph Company, Mexico City, Mexico

Compañía Peruana de Teléfonos Limitada, Lima, Peru

Porto Rico Telephone Company, San Juan, Puerto Rico

Shanghai Telephone Company, Federal Inc. U.S.A., Shanghai, China

Radiotelephone and Radiotelegraph Operating Companies

Compañía Internacional de Radio, Buenos Aires, Argentina

Compañía Internacional de Radio Boliviana, La Paz, Bolivia

Companhia Radio Internacional do Brasil, Rio de Janeiro, Brazil

Compañía Internacional de Radio, S.A., Santiago, Chile

Radio Corporation of Cuba, Havana, Cuba

Radio Corporation of Porto Rico, San Juan, Puerto Rico[1]

[1] Radiotelephone and radio broadcasting services.

Cable and Radiotelegraph Operating Companies

(Controlled by American Cable & Radio Corporation, New York, New York)

The Commercial Cable Company, New York, New York[2]

Mackay Radio and Telegraph Company, New York, New York[3]

All America Cables and Radio, Inc., New York, New York[4]

Sociedad Anónima Radio Argentina, Buenos Aires, Argentina[5]

[2] Cable service. [3] International and marine radiotelegraph services.
[4] Cable and radiotelegraph services. [5] Radiotelegraph service.

Laboratories

Federal Telecommunication Laboratories, Inc., Nutley, New Jersey

Standard Telecommunication Laboratories, Limited, London, England

Laboratoire Central de Télécommunications, Paris, France